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The Numerical Study of Efforts from the 4r Symmetrical Spherical Quadrilateral Mechanism with Technical Deviations

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The technological (geometrical) deviations determine in the intermediate couples of the cardan joint supplementary efforts due to restrained movement. The 4R spherical quadrilateral mechanism, where by R is notated the rotation kinematic pair, from which is obtained the cardan mechanism, is of third family, as consequence, multiple statically indeterminate and for calculating the reactions from the kinematic pairs it is applied the elastic linear calculation, using the relative displacements method noted in plukerian coordinates. The results of the numerical solving of this problem will be presented under the form of a diagrams and will be commented.

Keywords: Quadrilateral mechanism; elastic calculation; cardan joint; technical deviation.

1. INTRODUCTION

This mechanism presents interest because is the all mechanisms basis of the primitive cardan joints and reproduces, cinematically, the movement of a system with a cardan joint [1-4]. The 4R spherical quadrilateral mechanism is statically undetermined [1,4,5] and for calculating the reactions from the kinematic pairs it is applied the elastic linear calculation

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[6,7], using the relative displacements method noted in plukerian coordinates [8]. The technological (geometrical) deviations lead to further efforts in the kinematic pairs. These, in the local system of reference are constant [9,10] but in general system vary harmonic after certain law which depends on the values of the deviations. They harmonic ranging, these become further sources of excitation which lead to the modification of dynamic response of the cardan transmission. Knowing the shape of variation these forces we can evaluate the influence on the dynamic response (characteristic pulses). This paper only refers to the influence of technological (geometrical) deviations on the efforts from kinematic pairs. The obtained results allow us to conclude the influence of the technological (geometrical) deviations over the unitary efforts that appear in the kinematic pairs of the mechanism. In a future paper I will also present the influence of these forces on the dynamic response of cardan transmissions.

2. MATERIALS AND METHODS

2.1 Aspects Regarding Technical Deviations

During the manufacturing and montage process of cardan transmissions, at the component elements may inevitable appear deviations that influence their cinematic and dynamics characteristics. Next we will refer to the subassembly cardan joint, made of bracket entry 1, cardan cross 2 and bracket exit 3 (see Fig. 1).

The possible deviations that may appear during the manufacturing process may be:

a. The deviation from A - the deviation from perpendicularity and intersection of the axis of the tail bracket 1 towards the reaming axis, the deviation from nominal mounting position of the bearing from A, deviations noted with $\{\tilde{\Delta}_A\}$.
b. Bracket deviation at the joint 2 - the deviation from perpendicularity and intersection of the reaming axis 1 of cross 1 towards the axis of the tail, $\{\tilde{\Delta}_2^s\}$.
c. Bracket deviation at the joint 3 - the deviation from perpendicularity and intersection of the reaming axis 2 of cross 1 towards the axis of the tail, $\{\tilde{\Delta}_3^s\}$.
d. Cross deviation at the joint 2 - the deviation from perpendicularity and intersection of the axle 1 of cross 2 towards the other axis of the cross axles, $\{\tilde{\Delta}_2^d\}$.
e. Cross deviation at the joint 3 - the deviation from perpendicularity and intersection of the axle 2 of cross 2 towards the other axis of the cross axles, $\{\tilde{\Delta}_3^d\}$.
f. Cross deviation at the joint 5 - the deviation from perpendicularity and intersection of the axle 3 of cross 2 towards the other axis of the cross axles, $\{\tilde{\Delta}_5^s\}$.
g. Cross deviation at the joint 6 - the deviation from perpendicularity and intersection of the axle 4 of cross 2 towards the other axis of the cross axles, $\{\tilde{\Delta}_6^s\}$.
h. the deviation from perpendicularity and intersection of the reaming axis 1 of cross 2 towards the axis of the tail, $\{\tilde{\Delta}_5^d\}$.
i. the deviation from perpendicularity and intersection of the reaming axis 2 of cross 2 towards the axis of the tail, $\{\tilde{\Delta}_6^d\}$.
j. Deviation from D - the deviation from perpendicularity and intersection of the axis of the tail bracket 2 towards the reaming axis, the deviation from nominal mounting position of the bearing from D, $\{\tilde{\Delta}_8^s\}+\{\tilde{\Delta}_D\}$.

2.2 Mathematical Model

The mechanically speaking (static) , the 4R mechanism has unknown the reactions from the kinematic pairs A, 2, 3, 5, 6, 8 and also the moment from the joint A (see Fig. 1), in total 6+5x5=31 unknowns and 3 elements x 6=18 equations, 31-18=13 times statically undetermined [1,4,5].

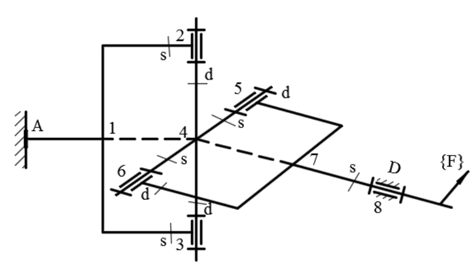


Fig. 1. The 4r symmetrical spherical quadrilateral mechanism

For determining these components a linear elastic calculation is used. In the following, a mathematical model with be elaborated for the linear elastic calculation of reactions, model that has as basis the method of relative displacements, presented in paper [8], with the notation in pluckerian coordinates. In the elastic calculation the joint from A is blocked (see Fig. 1), thing that explains the apparition as unknown of the axial moment in that point. The pivotal points 1, 4, 7, have the displacements $\{\Delta_1\}$, $\{\Delta_4\}$, $\{\Delta_7\}$, and the pivotal points with the kinematic pairs 2, 3, 5, 6 have left-right displacements with $\{\Delta_2^s\}$, $\{\Delta_2^d\}$, $\{\Delta_3^s\}$, $\{\Delta_3^d\}$, $\{\Delta_5^s\}$, $\{\Delta_5^d\}$, $\{\Delta_6^s\}$, $\{\Delta_6^d\}$. Pivotal point 8 has the displacement to left $\{\Delta_8^s\}$ and to right $\{\Delta_8^d\}=\{0\}$ .So the relations are written under the form:

$$
\begin{aligned}
&\{\Delta_2^d\}=\{\Delta_2^s\}+\xi_2\{U_2\}\\
&\{\Delta_3^d\}=\{\Delta_3^s\}+\xi_3\{U_3\}\\
&\{\Delta_5^d\}=\{\Delta_5^s\}+\xi_5\{U_5\}\\
&\{\Delta_6^d\}=\{\Delta_6^s\}+\xi_6\{U_6\}\\
&\{0\}=\{\Delta_8^s\}+\xi_7\{U_7\}.
\end{aligned}
\tag{1}
$$

From the pivotal points balance $[Kij]=[Kji]$.

$$
\begin{cases}
[K_{12}]\{\{\Delta_1\}-\{\Delta_2^s\}\}+[K_{1A}]\{\{\Delta_1\}-\{\Delta_A^0\}\}+[K_{13}]\{\{\Delta_1\}-\{\Delta_3^s\}\}=\{P_1\}\\
[K_{21}]\{\{\Delta_2^s\}-\{\Delta_1\}\}+[K_{24}]\{\{\Delta_2^d\}-\{\Delta_4\}\}=\{P_2\}\\
[K_{31}]\{\{\Delta_3^s\}-\{\Delta_1\}\}+[K_{34}]\{\{\Delta_3^d\}-\{\Delta_4\}\}=\{P_3\}\\
[K_{42}]\{\{\Delta_4\}-\{\Delta_2^d\}\}+[K_{43}]\{\{\Delta_4\}-\{\Delta_3^d\}\}+[K_{45}]\{\{\Delta_4\}-\{\Delta_5^s\}\}+[K_{46}]\{\{\Delta_4\}-\{\Delta_6^s\}\}=\{P_4\}\\
[K_{54}]\{\{\Delta_5^s\}-\{\Delta_4\}\}+[K_{57}]\{\{\Delta_5^d\}-\{\Delta_7\}\}=\{P_5\}\\
[K_{64}]\{\{\Delta_6^s\}-\{\Delta_4\}\}+[K_{67}]\{\{\Delta_6^d\}-\{\Delta_7\}\}=\{P_6\}\\
[K_{75}]\{\{\Delta_7\}-\{\Delta_5^d\}\}+[K_{76}]\{\{\Delta_7\}-\{\Delta_6^d\}\}+[K_{78}]\{\{\Delta_7\}-\{\Delta_8^s\}\}=\{P_7\}
\end{cases}
\tag{2}
$$

Are noted

$$
\begin{aligned}
&[K_{11}]=[K_{1A}]+[K_{12}]+[K_{13}]\\
&[K_{22}]=[K_{21}]+[K_{24}]\ ;\ [K_{33}]=[K_{31}]+[K_{34}]\\
&[K_{44}]=[K_{42}]+[K_{43}]+[K_{45}]+[K_{46}]\\
&[K_{55}]=[K_{54}]+[K_{57}]\ ;\ [K_{66}]=[K_{64}]+[K_{67}]\\
&[K_{77}]=[K_{75}]+[K_{76}]+[K_{78}].
\end{aligned}
\tag{3}
$$

and taking into account the relations (1) it results

$$
\begin{cases}
[K_{11}]\{\Delta_1\}-[K_{12}]\{\Delta_2^s\}-[K_{13}]\{\Delta_3^s\}=\{P_1\}\\
[K_{21}]\{\Delta_1\}+[K_{22}]\{\Delta_2^s\}-[K_{24}]\{\Delta_4\}+\xi_2[K_{24}]\{U_2\}=\{P_2\}\\
-[K_{31}]\{\Delta_1\}+[K_{33}]\{\Delta_3^s\}-[K_{34}]\{\Delta_4\}+\xi_3[K_{34}]\{U_3\}=\{P_3\}\\
-[K_{31}]\{\Delta_1\}+[K_{33}]\{\Delta_3^s\}-[K_{34}]\{\Delta_4\}+\xi_3[K_{34}]\{U_3\}=\{P_3\}\\
-[K_{54}]\{\Delta_4\}-[K_{55}]\{\Delta_5^s\}-[K_{57}]\{\Delta_7\}+\xi_5[K_{57}]\{U_5\}=\{P_5\}\\
-[K_{64}]\{\Delta_4\}+[K_{66}]\{\Delta_6^s\}-[K_{67}]\{\Delta_7\}+\xi_6[K_{67}]\{U_6\}=\{P_6\}\\
[K_{75}]\{\Delta_5^s\}-[K_{76}]\{\Delta_6^s\}+[K_{77}]\{\Delta_7\}-\xi_5[K_{75}]\{U_5\}-\xi_6[K_{76}]\{U_6\}+\xi_8[K_{78}]\{U_8\}=\{P_7\}
\end{cases}
\tag{4}
$$

where $\{P\}=[P_1,P_2,P_3,P_4,P_5,P_6,P_7]^T$ is the matrix of the forces that take action in the pivotal points, caused by the technical deviations, and is given by the relation:

$$\begin{cases} [K_{12}]\{\tilde{\Delta}_2^s\}+[K_{1A}]\{\tilde{\Delta}_A^s\}+[K_{13}]\{\tilde{\Delta}_3^s\}=\{P_1\} \\ -[K_{12}]\{\tilde{\Delta}_2^s\}-[K_{24}]\{\tilde{\Delta}_2^d\}=\{P_2\} \\ -[K_{13}]\{\tilde{\Delta}_3^s\}-[K_{34}]\{\tilde{\Delta}_3^d\}=\{P_3\} \\ [K_{24}]\{\tilde{\Delta}_2^d\}+[K_{34}]\{\tilde{\Delta}_3^d\}+[K_{45}]\{\tilde{\Delta}_5^s\}+[K_{46}]\{\tilde{\Delta}_6^s\}=\{P_4\} \\ -[K_{45}]\{\tilde{\Delta}_5^s\}-[K_{57}]\{\tilde{\Delta}_5^d\}=\{P_5\} \\ -[K_{46}]\{\tilde{\Delta}_6^s\}-[K_{67}]\{\tilde{\Delta}_6^d\}=\{P_6\} \\ [K_{57}]\{\tilde{\Delta}_5^d\}+[K_{67}]\{\tilde{\Delta}_6^d\}+[K_{78}]\left[\{\tilde{\Delta}_8^s\}-\{\tilde{\Delta}_D\}\right]=\{P_7\} \end{cases} \quad (5)$$

and

$$\{\tilde{\Delta}\}=\left[\{\tilde{\Delta}_A^s\}^T,\{\tilde{\Delta}_2^s\}^T,\{\tilde{\Delta}_2^d\}^T\{\tilde{\Delta}_3^s\}^T,\{\tilde{\Delta}_3^d\}^T,\{\tilde{\Delta}_5^s\}^T,\{\tilde{\Delta}_5^d\}^T,\{\tilde{\Delta}_6^s\}^T\{\tilde{\Delta}_6^d\}^T,\{\tilde{\Delta}_8^s\}^T,\{\tilde{\Delta}_D\}^T\right]^T \quad (6)$$

Is the matrix of technical deviations of the mechanism elements.

The notations are made as

$$\begin{aligned} &\{\Delta\}=\left[\{\Delta_1\}^T,\{\Delta_2^s\}^T,\{\Delta_3^s\}^T,\{\Delta_4\}^T,\{\Delta_5^s\}^T,\{\Delta_6^s\}^T,\{\Delta_7\}^T\right]^T \\ &\{\xi\}=[\xi_2,\xi_3,\xi_5,\xi_6,\xi_8] \\ &[V_2]=[1,0,0,0,0]\ ;\ [V_3]=[0,1,0,0,0]\ ;\ [V_5]=[0,0,1,0,0] \\ &[V_6]=[0,0,0,1,0]\ ;\ [V_8]=[0,0,0,0,1]. \end{aligned} \quad (7)$$

and then

$$\zeta_i=[V_i]\{\xi\}. \quad (8)$$

and $\zeta_2[K_{24}]\{U_2\}=[K_{24}]\{U_2\}[V_2]\{\xi\}$, and the analogue.

Also, the notations are made

$$[K_1]=\begin{bmatrix} [K_{11}] & -[K_{12}] & -[K_{13}] & [0] & [0] & [0] & [0] \\ -[K_{21}] & [K_{22}] & [0] & -[K_{24}] & [0] & [0] & [0] \\ -[K_{31}] & [0] & [K_{33}] & [K_{34}] & [0] & [0] & [0] \\ [0] & -[K_{42}] & -[K_{43}] & [K_{44}] & -[K_{45}] & -[K_{46}] & [0] \\ [0] & [0] & [0] & -[K_{54}] & [K_{55}] & [0] & -[K_{57}] \\ [0] & [0] & [0] & -[K_{64}] & [0] & [K_{66}] & -[K_{67}] \\ [0] & [0] & [0] & [0] & -[K_{75}] & -[K_{76}] & [K_{77}] \end{bmatrix}. \quad (9)$$

$$[K_2]=\begin{bmatrix}[0]\\ [K_{24}][U_2][V_2]\\ [K_{34}][U_3][V_3]\\ -[K_{24}][U_2][V_2]-[K_{34}][U_3][V_3]\\ [K_{57}][U_5][V_5]\\ [K_{67}][U_6][V_6]\\ -[K_{57}][U_5][V_5]-[K_{67}][U_6][V_6]+[K_{78}][U_8][V_8]\end{bmatrix}. \quad (10)$$

and then the equations (4) are combined into the equation

$$[K_1]\{\Delta\}+[K_2]\{\xi\}=\{P\}. \quad (11)$$

Equivalent with 42 scalar equations.

Isolating the left side of the pair 2, (see Fig. 2) results that

$$\{R_2\}=\{E_2^s\}=[K_{21}]\{\{\Delta_2^s\}-\{\Delta_1\}\} \quad (12)$$

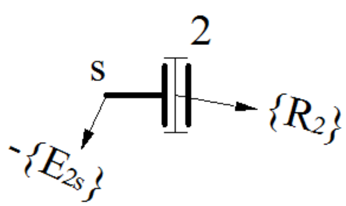


Fig. 2. The isolation of kinematic 2

and the analogue

$$\begin{aligned}\{R_3\}&=\{E_3^s\}=[K_{31}]\{\{\Delta_3^s\}-\{\Delta_1\}\}\\ \{R_5\}&=\{E_5^s\}=[K_{54}]\{\{\Delta_5^s\}-\{\Delta_4\}\}\\ \{R_6\}&=\{E_6^s\}=[K_{64}]\{\{\Delta_6^s\}-\{\Delta_4\}\}\\ \{R_8\}&=\{E_8^s\}=\{E\}=[K_{87}]\{\{\Delta_8^s\}-\{\Delta_7\}\}.\end{aligned} \quad (13)$$

as [8], $\{\tilde{U}_i\}^T\{R_i\}=0$ and $\{\Delta_8^s\}=-\xi_8\{U_8\}$ the equations are obtained

$$\begin{cases}\{\tilde{U}_2\}^T[K_{21}]\{\Delta_2^s\}-\{\tilde{U}_2\}^T[K_{21}]\{\Delta_1\}=Q_1\\ \{\tilde{U}_3\}^T[K_{31}]\{\Delta_3^s\}-\{\tilde{U}_3\}^T[K_{31}]\{\Delta_1\}=Q_2\\ \{\tilde{U}_5\}^T[K_{54}]\{\Delta_5^s\}-\{\tilde{U}_5\}^T[K_{54}]\{\Delta_4\}=Q_3\\ \{\tilde{U}_6\}^T[K_{64}]\{\Delta_6^s\}-\{\tilde{U}_6\}^T[K_{64}]\{\Delta_4\}=Q_4\\ \{\tilde{U}_8\}^T[K_{87}]\{\Delta_7\}+\xi_8\{\tilde{U}_8\}^T[K_{87}]\{U_8\}=Q_5\end{cases} \quad (14)$$

where the matrix $\{Q\}=[Q_1,Q_2,Q_3,Q_4,Q_5]^T$ is given by the relation:

$$\begin{cases}\{\tilde{U}_2\}^T[K_{21}]\{\tilde{\Delta}_2^s\}=Q_1\\ \{\tilde{U}_3\}^T[K_{31}]\{\tilde{\Delta}_3^s\}=Q_2\\ \{\tilde{U}_5\}^T[K_{54}]\{\tilde{\Delta}_5^s\}=Q_3\\ \{\tilde{U}_6\}^T[K_{64}]\{\tilde{\Delta}_6^s\}=Q_4\\ 0=Q_5\end{cases} \quad (15)$$

With the notations

$$[K_3]=\begin{bmatrix}\{\tilde{U}_2\}^T[K_{21}] & -\{\tilde{U}_2\}^T[K_{21}] & 0 & 0 & 0 & 0 & 0\\ \{\tilde{U}_3\}^T[K_{31}] & 0 & -\{\tilde{U}_3\}^T[K_{31}] & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \{\tilde{U}_5\}^T[K_{54}] & -\{\tilde{U}_5\}^T[K_{54}] & 0 & 0\\ 0 & 0 & 0 & \{\tilde{U}_6\}^T[K_{64}] & 0 & -\{\tilde{U}_6\}^T[K_{64}] & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \{\tilde{U}_8\}^T[K_{78}]\end{bmatrix} \quad (16)$$

$$[K_4]=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&\{\tilde{U}_8\}^T[K_{78}]\{U_8\}\end{bmatrix};\{Q\}=\begin{bmatrix}Q_1\\Q_2\\Q_3\\Q_4\\Q_5\end{bmatrix} \quad (17)$$

the equations (12),(13) are combined in the matrix equation

$$[K_3]\{\Delta\}+[K_4]\{\xi\}=\{Q\} \tag{18}$$

Equivalent with 5 scalar equations.

The equations (11), (18) can be narrowed with the notations

$$[K]=\begin{bmatrix}[K_1] & [K_2]\\ [K_3] & [K_4]\end{bmatrix}. \tag{19}$$

in the equation

$$[K]\begin{bmatrix}\{\Delta\}\\ \{\xi\}\end{bmatrix}=\begin{bmatrix}\{P\}\\ \{Q\}\end{bmatrix} \tag{20}$$

If the force $\{F\}$ occurs, then:

$$[K]\begin{bmatrix}\{\Delta\}\\ \{\xi\}\end{bmatrix}=\begin{bmatrix}\{P\}\\ \{Q\}+\{F\}\end{bmatrix} \tag{21}$$

equivalent with 47 equations with 47 unknowns from which results $\{\Delta\}$ and $\{\xi\}$, and the reactions and efforts are calculated with the relations (10), (11) in which $\{\Delta_8^s\}=-\xi_8\{U_8\}$. The deviations are then defined in one of the three reference systems OX_0Y^*Z, OX^*YZ, $OX^*YZ_0^*$ and that don't depend of θ_1,θ_2 and then in the system $OX_0Y_0Z_0$ and under this form the relations (5), (15) are used.

2.3 Calculation Algorithm

1. The indexation of bars is done from $1,2,...,14$ and the lengths are noted with l_i, $1,2,...,14$ and $l_2=l_4$; $l_3=l_5$; $l_6=l_7$; $l_8=l_9$; $l_{10}=l_{12}$; $l_{11}=l_{13}$.

2. The inertia moment is calculated:

$$I_{iy}=I_{iz}=\frac{\pi d_i^4}{64}\ ;I_{ix}=I_{iy}+I_{iz}\ ;A_i=\frac{\pi d_i^2}{4}. \tag{22}$$

3. The rigidity matrixes are calculated in the local reference system:

$$[k_i]=\begin{bmatrix}0 & 0 & 0 & \frac{E_iA_i}{l_i} & 0 & 0\\ 0 & 0 & \frac{6E_iI_{iz}}{l_i^2} & 0 & \frac{12E_iI_{iz}}{l_i^3} & 0\\ 0 & -\frac{6E_iI_{iy}}{l_i^2} & 0 & 0 & 0 & \frac{12E_iI_{iy}}{l_i^3}\\ \frac{G_iI_{ix}}{l_i} & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{4E_iI_{iy}}{l_i} & 0 & 0 & 0 & -\frac{6E_iI_{iy}}{l_i^2}\\ 0 & 0 & \frac{4E_iI_{iz}}{l_i} & 0 & \frac{6E_iI_{iz}}{l_i^2} & 0\end{bmatrix}. \tag{23}$$

$$[h_i]=\begin{bmatrix}0 & 0 & 0 & \frac{l_i}{G_iI_{ix}} & 0 & 0\\ 0 & 0 & \frac{l_i^2}{2E_iI_{iy}} & 0 & \frac{l_i}{E_iI_{iy}} & 0\\ 0 & -\frac{l_i^2}{2E_iI_{iz}} & 0 & 0 & 0 & \frac{l_i}{E_iI_{iz}}\\ \frac{l_i}{E_iA_i} & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{l_i^3}{3E_iI_{iz}} & 0 & 0 & 0 & -\frac{l_i^2}{2E_iI_{iz}}\\ 0 & 0 & \frac{l_i^3}{3E_iI_{iy}} & 0 & \frac{l_i^2}{2E_iI_{iy}} & 0\end{bmatrix}. \tag{24}$$

where, E_i,G_i are the elasticity modules

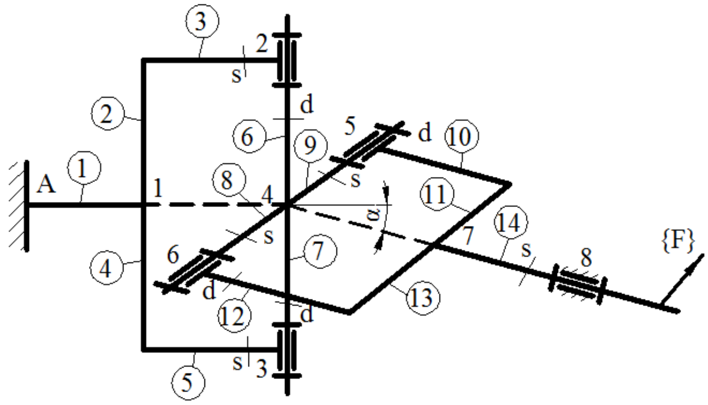


Fig. 3. Indexing bars

4. The matrix $[G_i]$; $[R_i]$ are calculated, with the relations from Table 1 and then the product $[G_i][R_i]$

$$[G_i]=\begin{bmatrix} 0 & -Z_i & Y_i \\ Z_i & 0 & -X_i \\ -Y_i & X_i & 0 \end{bmatrix}. \tag{25}$$

5. The position matrixes are calculated:

$$[T_i]=\begin{bmatrix} [R_i] & [0] \\ [G_i][R_i] & [R_i] \end{bmatrix} \; ; \; [T_i]^{-1}=\begin{bmatrix} [R_i]^T & [0] \\ [R_i]^T[G_i]^T & [R_i]^T \end{bmatrix}. \tag{26}$$

6. The matrixes $[H_i^*]$, $[K_i^*]$ are calculated:

$$[H_i^*]=[T_i][h_i][T_i]^{-1} \; ; \; [K_i^*]=[T_i][k_i][T_i]^{-1} \tag{27}$$

7. The matrixes $[H_{23}^*]$, $[H_{45}^*]$, $[H_{10,11}^*]$, $[H_{12,13}^*]$ are calculated:

$$\begin{gathered} [H_{23}^*]=[H_2^*]+[H_3^*] \; ; \; [H_{45}^*]=[H_4^*]+[H_5^*] \\ [H_{10,11}^*]=[H_{10}^*]+[H_{11}^*] \; ; \; [H_{12,13}^*]=[H_{12}^*]+[H_{13}^*]. \end{gathered} \tag{28}$$

8. It is identified:

$$\begin{gathered} [K_{1A}^*]=[K_{11}^*] \; ; \; [K_{12}^*]=[H_{23}^*]^{-1} ; \; [K_{13}^*]=[H_{45}^*]^{-1} \\ [K_{24}^*]=[K_6^*] \; ; \; [K_{34}^*]=[K_7^*] \; ; \; [K_{45}^*]=[K_9^*] \\ [K_{46}^*]=[K_8^*] \; ; \; [K_{57}^*]=[H_{10,11}^*]^{-1} ; \; [K_{67}^*]=[H_{12,13}^*]^{-1} \\ [K_{67}^*]=[H_{12,13}^*]^{-1} ; \; [K_{78}^*]=[K_{14}^*]. \end{gathered} \tag{29}$$

9. θ_2 is calculated with the formulas:

$$\theta_2 = \begin{cases} arctg(\frac{1}{c\alpha} tg\theta_1); 0 \le \theta_1 < \frac{\pi}{2} \\ \frac{\pi}{2}; \theta_1 = \frac{\pi}{2} \\ \pi + arctg(\frac{1}{c\alpha} tg\theta_1); \frac{\pi}{2} < \theta_1 < \frac{3\pi}{2} \\ \frac{3\pi}{2}; \theta_1 = \frac{3\pi}{2} \\ 2\pi + arctg(\frac{1}{c\alpha} tg\theta_1); \frac{3\pi}{2} < \theta_1 \le 2\pi \end{cases} . \quad (30)$$

10. $[T_{AB}],[T_{BC}],[T_{CD}]$ are calculated with the formulas:

$$[T_{AB}] = \begin{bmatrix} [R_{AB}] & [0] \\ [0] & [R_{AB}] \end{bmatrix} \quad ; \quad [T_{BC}] = \begin{bmatrix} [R_{BC}] & [0] \\ [0] & [R_{BC}] \end{bmatrix}$$
$$[T_{CD}] = \begin{bmatrix} [R_{CD}] & [0] \\ [0] & [R_{CD}] \end{bmatrix}. \quad (31)$$

Where

$$[R_{AB}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_1 & -s\theta_1 \\ 0 & s\theta_1 & c\theta_1 \end{bmatrix} \; ; \; [R_{BC}] = \begin{bmatrix} c\theta_1 c\theta_2 + s\theta_1 s\theta_2 c\alpha & s\alpha s\theta_2 & 0 \\ -c\theta_1 s\theta_2 s\alpha & c\theta_2 & -s\theta_1 \\ -s\theta_1 s\theta_2 s\alpha & c\alpha s\theta_2 & c\theta_1 \end{bmatrix}$$
$$[R_{CD}] = \begin{bmatrix} c\alpha & s\alpha s\theta_2 & s\alpha c\theta_2 \\ 0 & c\theta_2 & -s\theta_2 \\ -s\alpha & c\alpha s\theta_2 & c\alpha c\theta_2 \end{bmatrix}. \quad (32)$$

11. Are calculated:

$[K_{1A}],[K_{12}],[K_{13}]$ with the formula

$$[K] = [T_{AB}][K^*][T_{AB}]^{-1}. \quad (33)$$

$[K_{24}],[K_{34}],[K_{45}],[K_{46}]$ with formulas type

$$[K] = [T_{BC}][K^*][T_{BC}]^{-1}. \quad (34)$$

$[K_{57}],[K_{67}],[K_{78}]$ with formulas type

$$[K] = [T_{CD}][K^*][T_{CD}]^{-1}. \quad (35)$$

12. Are calculated:

$$\begin{aligned}&\{U_2\}=\{U_3\}=[0,-s\theta_1,c\theta_1,0,0,0]^T\\&\{U_5\}=\{U_6\}=[s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha,0,0,0]^T\\&\{U_8\}=[c\alpha,0,-s\alpha,0,0,0]^T.\end{aligned}\tag{36}$$

$$\begin{aligned}&\{\tilde{U}_2\}=\{\tilde{U}_3\}=[0,0,0,0,-s\theta_1,c\theta_1]^T\\&\{\tilde{U}_5\}=\{\tilde{U}_6\}=[0,0,0,s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha]^T\\&\{\tilde{U}_8\}=[0,0,0,c\alpha,0,-s\alpha]^T.\end{aligned}\tag{37}$$

$$\{F\}=\tilde{M}[0,0,0,c\alpha,0,-s\alpha]^T \;\; ; \;\; \{\tilde{U}_8\}\{F\}=\tilde{M}.\tag{38}$$

13. For a given θ_1 and

$$\{\Delta\}=\left[\{\Delta_A^l\}^T,\{\Delta_{2s}^l\}^T,\{\Delta_{2d}^l\}^T\{\Delta_{3s}^l\}^T,\{\Delta_{3d}^l\}^T,\{\Delta_{5s}^l\}^T,\{\Delta_{5d}^l\}^T,\{\Delta_{6s}^l\}^T\{\Delta_{6d}^l\}^T,\{\Delta_{8s}^l\}^T,\{\Delta_D^l\}^T\right]^T$$

chosen, $\{\tilde{\Delta}\}=\left[\{\tilde{\Delta}_A^s\}^T,\{\tilde{\Delta}_2^s\}^T,\{\tilde{\Delta}_2^d\}^T\{\tilde{\Delta}_3^s\}^T,\{\tilde{\Delta}_3^d\}^T,\{\tilde{\Delta}_5^s\}^T,\{\tilde{\Delta}_5^d\}^T,\{\tilde{\Delta}_6^s\}^T\{\tilde{\Delta}_6^d\}^T,\{\tilde{\Delta}_8^s\}^T,\{\tilde{\Delta}_D\}^T\right]^T$

is calculated with the formulas from Table 2.

14. Are calculated the matrixes $\{P\}=[P_1,P_2,P_3,P_4,P_5,P_6,P_7]^T$ and $\{Q\}=[Q_1,Q_2,Q_3,Q_4,Q_5]^T$ with the reactions (5), (15).

15. Are calculated the matrixes $[K_1],[K_2],[K_3],[K_4],\{P\},\{Q\},[K]$ with the formulas (9), (10), (16), (17), (5),(15)),(19).

16. It is solved the matrix equation (20), (21) in case of need.

17. The reaction from A is calculated with the relation

$$\{R_A\}=-[K_{1A}]\{\Delta_1\}.\tag{39}$$

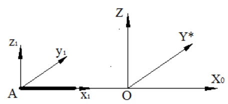
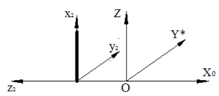
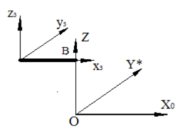
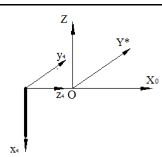
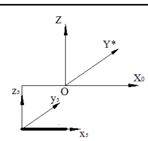
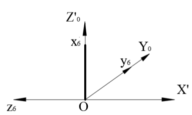
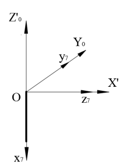
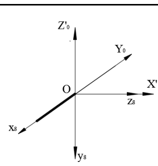
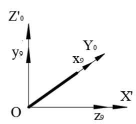
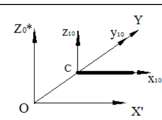
and expressed in local coordinates.

18. The reactions are calculated $\{R_2\},\{R_3\},\{R_5\},\{R_6\},\{R_8\}$ with the relations (12), (13).

19. The reactions are expressed under $\{R_2\},\{R_3\},\{R_5\},\{R_6\},\{R_8\}$ in local system coordinates.

20. The graphs $\xi_2,\xi_3,\xi_5,\xi_6,\xi_8,R_{1x},R_{1y},R_{1z},M_{1x},M_{1y},M_{1z}$, $R_{2x},R_{2y},R_{2z},M_{2x},M_{2y},M_{2z}$, $R_{3x},R_{3y},R_{3z},M_{3x},M_{3y},M_{3z}$, $R_{5x},R_{5y},R_{5z},M_{5x},M_{5y},M_{5z}$, $R_{6x},R_{6y},R_{6z},M_{6x},M_{6y},M_{6z}$, $R_{8x},R_{8y},R_{8z},M_{8x},M_{8y},M_{8z}$.
are made taking into account θ_1.

Table 1. The expressions of the position and rotation matrixes $[G_i],[R_i]$

The element	The local reference system $Ox_i y_i z_i$	The coordinates of the local reference system X_i, Y_i, Z_i	The position matrix $[G_i]$	The rotation matrix $[R_i]$
1		$X_1 = -(l_1 + l_3)$ $Y_1 = 0$ $Z_1 = 0$	$[G_1] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_1+l_3 \\ 0 & -(l_1+l_3) & 0 \end{bmatrix}$	$[R_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2		$X_2 = -l_3$ $Y_2 = 0$ $Z_1 = 0$	$[G_2] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_2] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
3		$X_3 = -l_3$ $Y_3 = 0$ $Z_3 = l_2$	$[G_3] = \begin{bmatrix} 0 & -l_2 & 0 \\ l_2 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4		$X_4 = -l_3$ $Y_4 = 0$ $Z_4 = 0$	$[G_4] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_4] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
5		$X_5 = -l_3$ $Y_5 = 0$ $Z_5 = -l_4$	$[G_5] = \begin{bmatrix} 0 & l_4 & 0 \\ -l_4 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_5] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
6		$X_6 = 0$ $Y_6 = 0$ $Z_6 = 0$	$[G_6] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_6] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
7		$X_7 = 0$ $Y_7 = 0$ $Z_7 = 0$	$[G_7] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_7] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
8		$X_8 = 0$ $Y_8 = 0$ $Z_8 = 0$	$[G_8] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_8] = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
9		$X_9 = 0$ $Y_9 = 0$ $Z_9 = 0$	$[G_9] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_9] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
10		$X_{10} = 0$ $Y_{10} = l_9$ $Z_{10} = 0$	$[G_{10}] = \begin{bmatrix} 0 & 0 & l_9 \\ 0 & 0 & 0 \\ -l_9 & 0 & 0 \end{bmatrix}$	$[R_{10}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

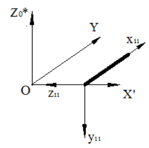
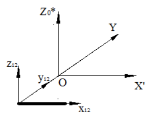
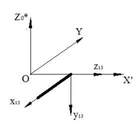
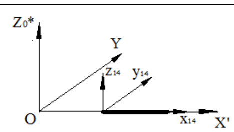
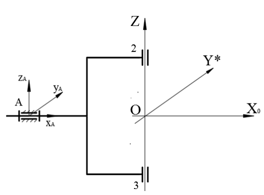
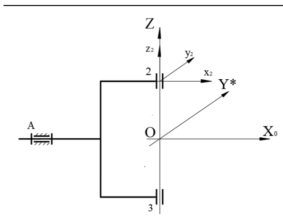
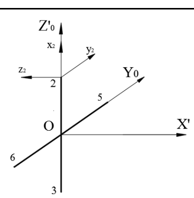
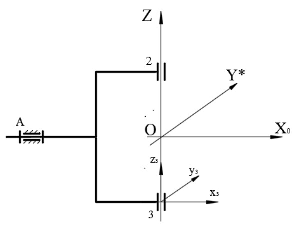
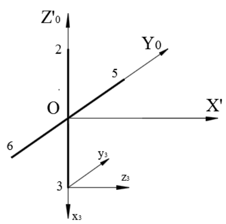
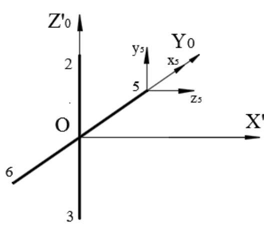
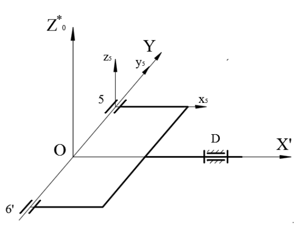
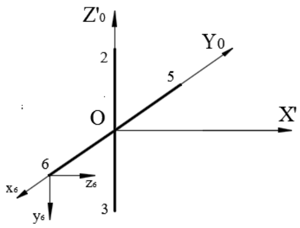
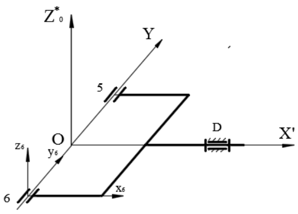
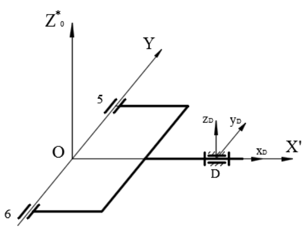
The element	**The local reference system** $Ox_i y_i z_i$	**The coordinates of the local reference system** X_i, Y_i, Z_i	**The position matrix** $[G_i]$	**The rotation matrix** $[R_i]$
11		$X_{11} = l_{10}$ $Y_{11} = 0$ $Z_{11} = 0$	$[G_{11}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_{10} \\ 0 & l_{10} & 0 \end{bmatrix}$	$[R_{11}] = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
12		$X_{12} = 0$ $Y_{12} = -l_8$ $Z_{12} = 0$	$[G_{12}] = \begin{bmatrix} 0 & 0 & -l_8 \\ 0 & 0 & 0 \\ l_8 & 0 & 0 \end{bmatrix}$	$[R_{12}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
13		$X_{13} = l_{12}$ $Y_{13} = 0$ $Z_{13} = 0$	$[G_{13}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_{12} \\ 0 & l_{12} & 0 \end{bmatrix}$	$[R_{13}] = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
14		$X_{14} = l_{10}$ $Y_{14} = 0$ $Z_{14} = 0$	$[G_{14}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_{10} \\ 0 & l_{10} & 0 \end{bmatrix}$	$[R_{14}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Table 2. The expressions of the geometrical deviations

$\{\tilde{\Delta}_A\}$ **the displacement of the bearing from A and the displacement of crosses tail 1**	$\{\Delta_A^l\} = [\theta_{Ax}^l, \theta_{Ay}^l, \theta_{Az}^l, \delta_{Ax}^l, \delta_{Ay}^l, \delta_{Az}^l]^T$
	$X_1 = -(l_1 + l_3)$; $Y_1 = 0$; $Z_1 = 0$ $[G_A] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_1 + l_3 \\ 0 & -(l_1 + l_3) & 0 \end{bmatrix}$; $[R_A] = [I]$ $\{\Delta_A\} = \begin{bmatrix} [I] & [0] \\ [G_A] & [I] \end{bmatrix} [\Delta_A^l]$; $\{\tilde{\Delta}_A\} = [T_{AB}]\{\Delta_A\}$
$\{\tilde{\Delta}_2^s\}$ the bracket displacement 1 at the joint 2	$\{\Delta_{2s}^l\} = [\theta_{2sx}^l, \theta_{2sy}^l, \theta_{2sz}^l, \delta_{2sx}^l, \delta_{2sy}^l, \delta_{2sz}^l]^T$
	$X_{2s} = 0$; $Y_{2s} = 0$; $Z_{2s} = l_6$ $[G_{2s}] = \begin{bmatrix} 0 & -l_6 & 0 \\ l_6 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $[R_{2s}] = [I]$ $\{\Delta_{2s}\} = \begin{bmatrix} [I] & [0] \\ [G_{2s}] & [I] \end{bmatrix} [\Delta_{2s}^l]$; $\{\tilde{\Delta}_{2s}\} = [T_{AB}]\{\Delta_{2s}\}$
$\{\tilde{\Delta}_2^d\}$ the cross displacement at the joint 2	$\{\Delta_{2d}^l\} = [\theta_{2dx}^l, \theta_{2dy}^l, \theta_{2dz}^l, \delta_{2dx}^l, \delta_{2dy}^l, \delta_{2dz}^l]^T$
	$X_{2d} = 0$; $Y_{2d} = 0$; $Z_{2d} = l_6$ $[G_{2d}] = \begin{bmatrix} 0 & -l_6 & 0 \\ l_6 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $[R_{2d}] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\{\tilde{\Delta}_A\}$ **the displacement of the bearing from A and the displacement of crosses tail 1**	$\{\Delta_A^l\} = [\theta_{Ax}^l, \theta_{Ay}^l, \theta_{Az}^l, \delta_{Ax}^l, \delta_{Ay}^l, \delta_{Az}^l]^T$
	$\{\Delta_{2d}\} = \begin{bmatrix} [R_{2d}] & [0] \\ [G_{2d}][R_{2d}] & [R_{2d}] \end{bmatrix} [\Delta_{2d}^l]$; $\{\tilde{\Delta}_{2d}\} = [T_{BC}]\{\Delta_{2d}\}$
$\{\tilde{\Delta}_3^s\}$ the cross displacement 1 at the joint 3	$\{\Delta_{3s}^l\} = [\theta_{3sx}^l, \theta_{3sy}^l, \theta_{3sz}^l, \delta_{3sx}^l, \delta_{3sy}^l, \delta_{3sz}^l]^T$
	$X_{3s} = 0$; $Y_{3s} = 0$; $Z_{3s} = -l_7$ $[G_{3s}] = \begin{bmatrix} 0 & l_7 & 0 \\ -l_7 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $[R_{3s}] = [I]$ $\{\Delta_{3s}\} = \begin{bmatrix} [I] & [0] \\ [G_{3s}] & [I] \end{bmatrix} [\Delta_{3s}^l]$; $\{\tilde{\Delta}_{3s}\} = [T_{AB}]\{\Delta_{3s}\}$
$\{\tilde{\Delta}_3^d\}$ the cross displacement at the joint 3	$\{\Delta_{3d}^l\} = [\theta_{3dx}^l, \theta_{3dy}^l, \theta_{3dz}^l, \delta_{3dx}^l, \delta_{3dy}^l, \delta_{3dz}^l]^T$
	$X_{3d} = 0$; $Y_{3d} = 0$; $Z_{3d} = -l_7$ $[G_{3d}] = \begin{bmatrix} 0 & l_7 & 0 \\ -l_7 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $[R_{3d}] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ $\{\Delta_{3d}\} = \begin{bmatrix} [R_{3d}] & [0] \\ [G_{3d}][R_{3d}] & [R_{3d}] \end{bmatrix} [\Delta_{3d}^l]$; $\{\tilde{\Delta}_{3d}\} = [T_{BC}]\{\Delta_{3d}\}$
$\{\tilde{\Delta}_5^s\}$ the cross displacement at the joint 5	$\{\Delta_{5s}^l\} = [\theta_{5sx}^l, \theta_{5sy}^l, \theta_{5sz}^l, \delta_{5sx}^l, \delta_{5sy}^l, \delta_{5sz}^l]^T$
	$X_{5s} = 0$; $Y_{5s} = l_9$; $Z_{5s} = 0$ $[G_{5s}] = \begin{bmatrix} 0 & 0 & l_9 \\ 0 & 0 & 0 \\ -l_9 & 0 & 0 \end{bmatrix}$; $[R_{5s}] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\{\Delta_{5s}\} = \begin{bmatrix} [R_{5s}] & [0] \\ [G_{5s}][R_{5s}] & [R_{5s}] \end{bmatrix} [\Delta_{5s}^l]$; $\{\tilde{\Delta}_{5s}\} = [T_{BC}]\{\Delta_{5s}\}$
$\{\tilde{\Delta}_5^d\}$ the bracket displacement 2 at the joint 5	$\{\Delta_{5d}^l\} = [\theta_{5dx}^l, \theta_{5dy}^l, \theta_{5dz}^l, \delta_{5dx}^l, \delta_{5dy}^l, \delta_{5dz}^l]^T$
	$X_{5d} = 0$; $Y_{5d} = l_9$; $Z_{5d} = 0$ $[G_{5d}] = \begin{bmatrix} 0 & 0 & l_9 \\ 0 & 0 & 0 \\ -l_9 & 0 & 0 \end{bmatrix}$; $[R_{5d}] = [I]$ $\{\Delta_{5s}\} = \begin{bmatrix} [I] & [0] \\ [G_{5d}] & [I] \end{bmatrix} [\Delta_{5d}^l]$; $\{\tilde{\Delta}_{5d}\} = [T_{CD}]\{\Delta_{5d}\}$
$\{\tilde{\Delta}_6^s\}$ the cross displacement at the joint 6	$\{\Delta_{6s}^l\} = [\theta_{6sx}^l, \theta_{6sy}^l, \theta_{6sz}^l, \delta_{6sx}^l, \delta_{6sy}^l, \delta_{6sz}^l]^T$

$\{\tilde{\Delta}_A\}$ **the displacement of the bearing from A and the displacement of crosses tail 1**	$\{\Delta_A^l\} = [\theta_{Ax}^l, \theta_{Ay}^l, \theta_{Az}^l, \delta_{Ax}^l, \delta_{Ay}^l, \delta_{Az}^l]^T$
	$X_{6s} = 0 \; ; Y_{6s} = -l_8 \; ; Z_{6s} = 0$ $[G_{6s}] = \begin{bmatrix} 0 & 0 & -l_8 \\ 0 & 0 & 0 \\ l_8 & 0 & 0 \end{bmatrix} \; ; \; [R_{6s}] = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ $\{\Delta_{6s}\} = \begin{bmatrix} [R_{6s}] & [0] \\ [G_{6s}][R_{6s}] & [R_{6s}] \end{bmatrix} [\Delta_{6s}^l] \; ; \; \{\tilde{\Delta}_{6s}\} = [T_{BC}]\{\Delta_{6s}\}$
$\{\tilde{\Delta}_6^d\}$ the bracket displacement 2 at the joint 6	$\{\Delta_{6d}^l\} = [\theta_{6dx}^l, \theta_{6dy}^l, \theta_{6dz}^l, \delta_{6dx}^l, \delta_{6dy}^l, \delta_{6dz}^l]^T$
	$X_{6d} = 0 \; ; Y_{6d} = -l_8 \; ; Z_{6d} = 0$ $[G_{6d}] = \begin{bmatrix} 0 & 0 & -l_8 \\ 0 & 0 & 0 \\ l_8 & 0 & 0 \end{bmatrix} \; ; \; [R_{6d}] = [I]$ $\{\Delta_{6d}\} = \begin{bmatrix} [I] & [0] \\ [G_{6d}] & [I] \end{bmatrix} [\Delta_{6d}^l] \; ; \; \{\tilde{\Delta}_{6d}\} = [T_{CD}]\{\Delta_{6d}\}$
$\{\tilde{\Delta}_8^s\}$ the displacement of the bracket axle 2	$\{\Delta_{8s}^l\} = [\theta_{8sx}^l, \theta_{8sy}^l, \theta_{8sz}^l, \delta_{8sx}^l, \delta_{8sy}^l, \delta_{8sz}^l]^T$
	$X_{8s} = l_{10} + l_{14} \; ; Y_{8s} = 0 \; ; Z_{8s} = 0$ $[G_{8s}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -(l_{10}+l_{14}) \\ l_8 & (l_{10}+l_{14}) & 0 \end{bmatrix} ; \; [R_{8s}] = [I]$ $\{\Delta_{8s}\} = \begin{bmatrix} [I] & [0] \\ [G_{8s}] & [I] \end{bmatrix} [\Delta_{8s}^l] \; ; \; \{\tilde{\Delta}_{8s}\} = [T_{CD}]\{\Delta_{8s}\}$

3. RESULTS AND DISCUSSION

It is considered the 4r symmetrical spherical quadrilateral mechanism from Fig. 3. Elements of the mechanism, have been indexed with the numbers $i = 1,2,...,14$ and are considered to have the lengths l_i , the diameter d_i , the elasticity modules E_i, G_i , the sections A_i and the main central inertial moments I_{iy}, I_{iz} , $I_{ix} = I_{iy} + I_{iz}$, $i = 1,2,...,14$ that has a technological (geometrical) deviation at the cardan cross.A shaft of cardan cross is longer with 0,0001 m than the nominal length. So $l_i = l = 0{,}05m$; $d_i = d = 0{,}02m$; $i = 1,2,...,14$; $E = 2{,}1 \cdot 10^{11} N/m^2$; $G = 8{,}1 \cdot 10^{10} N/m^2$; $\delta_{2x} = 0{,}0001m$ (see Fig. 4).

This deviation is defined in the local reference system and is expressed in the plukerian coordinate under $\{\Delta_2^l\} = [0, \; 0, \; 0, 0.0001, \; 0, \; 0]^T$.

To solve this problem may use any conventional method of calculation numerical [11,12].On the basis of a calculation program realized in Excel with the algorithm presented in this paper, where obtained, in the case where $\alpha = 0^\circ$, for the reactions from the kinematic pair 2 and 3, $R_{2X}^0, R_{2Y}^0, R_{2Z}^0, R_{3X}^0, R_{3Y}^0, R_{3Z}^0$, the results from diagrams from Fig. 5.

For the reactions $R_{2X}^0, R_{2Y}^0, R_{2Z}^0, R_{3X}^0, R_{3Y}^0, R_{3Z}^0$, from the own reference system, the results are in the diagrams from Fig. 6.

It is found as expected that in the local reference systems the reactions forces are constant and this is due to the invariable positioning towards these systems. For the reaction from the joints 5 and 6, in general reference systems, are obtained the values presented in Fig. 7.

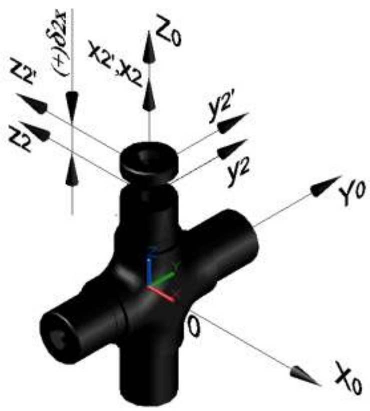


Fig. 4. Technical deviation at joints 2 in local reference system

and in own reference systems values presented in Fig. 8.

Analogue, for the reactions from the other joints, in the own reference systems, are obtained constant values. For α≠0°, in the own reference system, the components $R^0_{2X}, R^0_{2Y}, R^0_{2Z}, R^0_{3X}, R^0_{3Y}, R^0_{3Z}$, $R^0_{5X}, R^0_{5Y}, R^0_{5Z}, R^0_{6X}, R^0_{6Y}, R^0_{6Z}$, aren't constant and have a variation depending on the angle θ_1. The resuts will be presented in next paper.

In Fig. 9 are presented the results obtained for reaction forces from kinematic pairs 2,3,5 and 6 of cardan cross in the case where $\alpha = 0^\circ$.

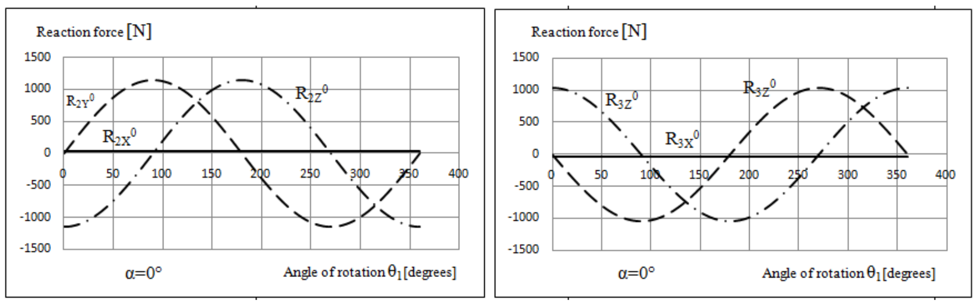


Fig. 5. The variation of reaction forces in joints 2 and 3 for *α=0°* in general reference system

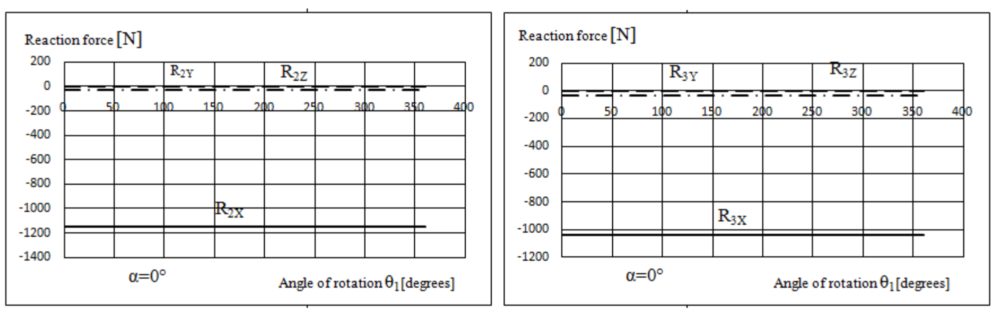


Fig. 6. The variation of reaction forces in joints 2 and 3 for *α=0°* in local reference system

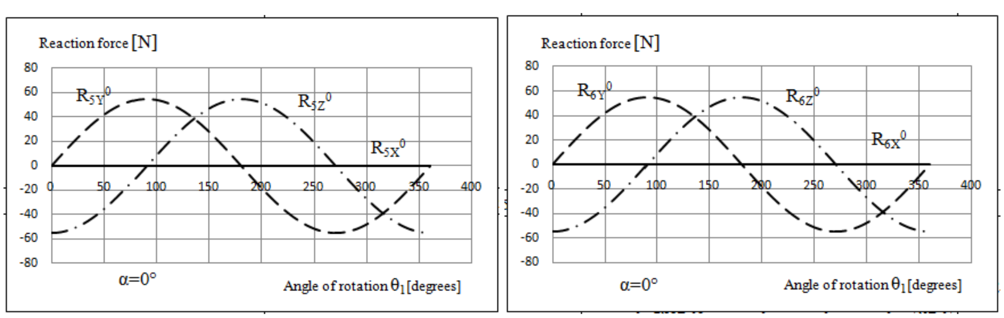


Fig. 7. The variation of reaction forces in joints 5 and 6 for *α=0°* in general reference system

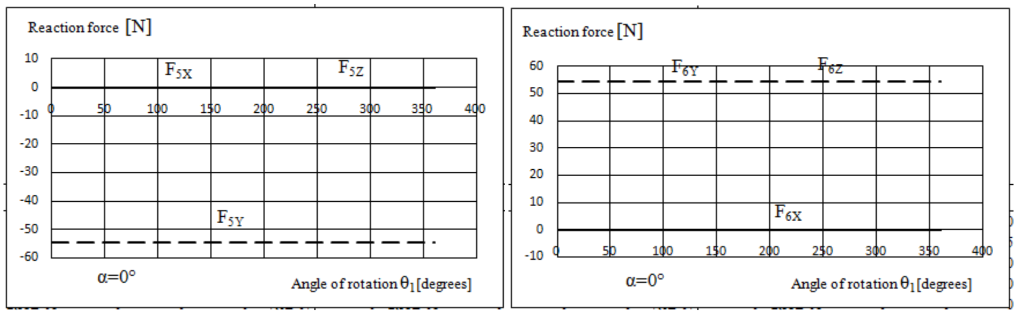


Fig. 8. The variation of reaction forces in joints 5 and 6 for *α=0°* in local reference system

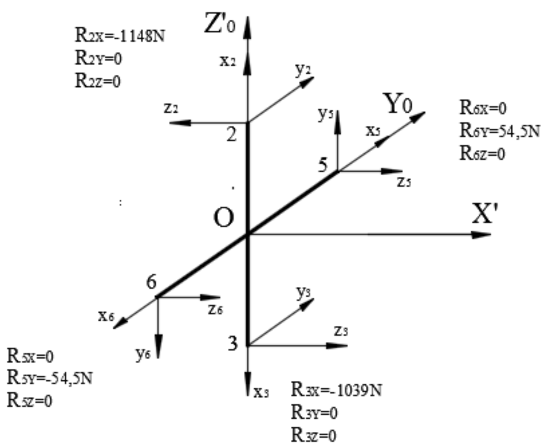


Fig. 9. The reaction forces in joints cardan cross for *α=0°* in local reference system

4. CONCLUSION

The elastic calculation and the relative displacements method makes possible the determination of the reactions from the kinematic pairs of the 4R spherical quadrilateral mechanism, that is multiple statically indeterminate.Following the numerical simulations made with the program realized using the mathematical model from the present paper, the following conclusions can be taken.

1. For α=0°, in the general reference system $OX_0Y_0Z_0$ the reaction forces vary with the angle θ_1, and in the local reference system they remain constant, depending only on the mechanical and geometrical characteristics of the element.
2. For α≠0°, both in local and general reference systems, the reactions and moments vary with the angle θ_1, as will show in next paper.
3. The reaction forces are acting on the direction of displacements and are increasing as the displacements increase.
4. It is found as expected that the resultant forces from mechanism is zero when the mechanism is not requested of external forces as shown in Fig. 9.

The results of this paper can be considered a point of departure in the journey of designing policardan transmissions for:

- Determining the influence of different technical deviations that appear inevitably during the fabrication, over the efforts from those elements,
- Optimizing the production tolerances and creating a montage for making production cheaper,
- Creating norms and standards regarding the production and montage tolerances of the component elements.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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