Asian Research Journal of Mathematics

12(3): 1-20, 2019; Article no.ARJOM.43604
ISSN: 2456-477X

Entropy Generation Analysis of a Reactive MHD Third Grade Fluid in a Cylindrical Pipe with Radially Applied Magnetic Field and Hall Current

Y. M. Aiyesimi[1], M. Jiya[2], G. A. Bolarin[2] and O. V. Akinremi[3*]

[1]*Department of Computer Sciences, Edo University, Iyamho, Edo State, Nigeria.*
[2]*Department of Mathematics, Federal University of Technology, Minna, Niger State, Nigeria.*
[3]*Department of Mathematics, Adeyemi College of Education, Ondo State, Nigeria.*

Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2019/v12i330086
Editor(s):
(1) Dr. Krasimir Yankov Yordzhev, Associate Professor, Faculty of Mathematics and Natural Sciences, South-West University, Blagoevgrad, Bulgaria.
Reviewers:
(1) Taza Gul, City University of Science and Information Technology (CUSIT), Pakistan.
(2) Grienggrai Rajchakit, Maejo University, Thailand.
(3) M. VeeraKrishna, Rayalaseema University, India.
(4) J. C. Leong, National Pingtung University of Science and Technology, Taiwan.
Complete Peer review History: http://www.sdiarticle3.com/review-history/43604

Original Research Article

Received: 18 October 2018
Accepted: 10 January 2019
Published: 01 March 2019

Abstract

The combined effects of chemical reaction, radially applied magnetic field and Hall effect on entropy generation of a steady third grade magnetohydrodynamic fluid flowing through a uniformly circular pipe was studied. The governing equations are presented and the resulting non-linear dimensionless equations are solved numerically using Galerkin Weighted Residual Method. The velocity, temperature and concentration profile were obtained and utilized in computing the entropy number. A parametric study of germane parameters involved are presented graphically and discussed. It was observed that irreversibility due to heat transfer dominates the flow compared to fluid friction and Hall parameter inhibits the Bejan number while Magnetic parameter enhances the Bejan number.

Keywords: Magnetohydrodynamic; Hall current; Galerkin Weighted Residual Method; Bejan number.

**Corresponding author: E-mail: akinremibv@gmail.com, akinremiov@aceondo.edu.ng;*

1 Introduction

Magnetohydrodynamic (MHD) flows in rectangular and cylindrical system continue to stimulate significant interest in the field of engineering science and applied mathematics. This interest is owned to the numerous important applications in biological and engineering industry such as reactive polymer flows, extraction of crude oil, synthetic fibres, paper production and also in absorption and filtration processes in chemical engineering. Krishna and Gangadhar Reddy [1] discussed the unsteady MHD free convection in a boundary layer flow of an electrically conducting fluid through porous medium subject to uniform transverse magnetic field over a moving infinite vertical plate in the presence of heat source and chemical reaction. Krishna and Subba Reddy [2] have investigated the simulation on the MHD forced convective flow through stumpy permeable porous medium (oil sands, sand) using Lattice Boltzmann method. Krishna and Jyothi [3] discussed the Hall effects on MHD Rotating flow of a visco-elastic fluid through a porous medium over an infinite oscillating porous plate with heat source and chemical reaction. Reddy et al. [4] investigated MHD flow of viscous incompressible nano-fluid through a saturating porous medium. Recently, Krishna et al. [5-8] discussed the MHD flows of an incompressible and electrically conducting fluid in planar channel. Veera Krishna et al. [9] discussed heat and mass transfer on unsteady MHD oscillatory flow of blood through porous arteriole. The effects of radiation and Hall current on an unsteady MHD free convective flow in a vertical channel filled with a porous medium have been studied by Veera Krishna et al. [10]. The heat generation/absorption and thermo-diffusion on an unsteady free convective MHD flow of radiating and chemically reactive second grade fluid near an infinite vertical plate through a porous medium and taking the Hall current into account have been studied by Veera Krishna and Chamkha [11]. Taza et al. [12] considered the heat transfer analysis in MHD thin film flow of third grade fluid on a vertical belt with slip boundary conditions. Veera Krishna et al. [13] investigated the heat and mass transfer on MHD free convective flow over an infinite non-conducting vertical flat porous plate. Veera Krishna and Jyothi [14] discussed the effect of heat and mass transfer on free convective rotating flow of a visco-elastic incompressible electrically conducting fluid past a vertical porous plate with time dependent oscillatory permeability and suction in presence of a uniform transverse magnetic field and heat source. Taza et al. [15] presented the analysis of a thin film flow in MHD third grade fluid past a vertical belt with temperature dependent viscosity emploring the ADM and OHAM.

The steady flow of a reactive variable viscosity fluid in a cylindrical pipe with isothermal wall was studied by Makinde [16], reporting the dependence of the steady state thermal ignition criticality conditions on both Frank-Kamenetskii and viscous heating parameters. Makinde et al. [17], numerical investigation for the entropy generation rates in an unsteady flow of a variable viscosity incompressible fluid through a porous pipe with uniform suction at the surface were examined. In Ajadi [18], closed-form solution using Homotopy Analysis method on the effect of variable viscosity and viscous dissipation on the thermal stability of a one-step exothermic reactive non-Newtonian flow in a cylindrical pipe assuming negligible reactant consumption were obtained. In, Aiyesimi et al. [19] considered a mathematical model for a dusty viscoelastic fluid flow in a circular channel was considered, observing that an increase in the value of magnetic field and viscoelastic parameter reduces the horizontal velocity of the fluid and particles, thereby reducing the boundary layer thickness, hence inducing an increase in the absolute value of the velocity gradient at the surface.

The thermodynamics second law analysis and its design-related concept of entropy generation minimization has been a cornerstone in the field transfer and thermal design. Several researchers were motivated to study fundamental and applied engineering problem based on second law analyses, due to the production of entropy resulting from combined effects of velocity and temperature gradient. Generating entropy is tied to thermodynamic irreversibility, which is common in all heat transfer process. Eegunjobi & Makinde [20] investigated the combined effects of buoyancy force and Navier slip on the entropy generation rate in a vertical porous channel with wall suction/injection. The combined effects of Navier slip, convective cooling, variable viscosity and suction/injection on the entropy generation rate in an unsteady flow of an incompressible viscous fluid flowing through a channel with permeable wall was studied by Chinyoka & Makinde [21].

In this paper, the motivation comes from a desire to gain more understanding into the combined effect of radially applied magnetic field and Hall current on the flow of chemically reactive third grade fluid. The relevant governing equation has been solved numerically by Galerkin Weighted Residual Method [22,23]. The effects of the various apposite parameters on the velocity, temperature and concentration are presented. In this work, entropy generation rate of a laminar MHD flow of a reactive third grade fluid is considered in a circular pipe, which is assumed electrically conducting and incompressible in the presence of an externally applied radially exponential magnetic field.

2 Mathematical Formulation

Considering a steady flow of electrically conducting, incompressible, third grade fluid in a non-conducting circular pipe in the absence of gravitational force. The z-axis is taken along the axis of flow. Radially exponential varying magnetic field $B_r = B_0 e^{\frac{r}{2R}}$ is applied (Bartella et al. [24]) and no electric field is applied. The flow is induced due to constant applied pressure gradient in the z-direction and electron atom collision frequency is assumed to be relatively high compared to the collision frequency of ions. The equations which govern the MHD flow are the continuity, momentum and Maxwell equations. In fluid dynamics studies, it is assumed that the flows meet the Clausius-Duhem inequality and the specific Helmholtz free energy of fluid has a minimum at equilibrium (Rajagopal, [25]). Using the velocity field $V = (0,\ 0,\ \mathrm{w}(r))$, the incompressibility condition is satisfied identically and momentum and Maxwell equations after the constitutive equations

$$\begin{aligned} T &= -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 \left(A_2 A_1 + A_1 A_2 \right) + \beta_3 (trA_1^2) A_1 \\ &\quad + \gamma_1 A_4 + \gamma_2 (A_3 A_1 + A_1 A_3) + \gamma_3 A_2^2 + \gamma_4 (A_2 A_1^2 + A_1^2 A_2) + \gamma_5 (trA_2) A_2 \\ &\quad + \gamma_6 (trA_2) A_1^2 + \left(\gamma_7 trA_3 + \gamma_8 tr(A_2 A_1) \right) A_1, \\ A_1 &= gradV + (gradV)^T \\ A_n &= \frac{dA_{n-1}}{dt} + A_{n-1} L + L^T A_{n-1}, \qquad (n > 1) \end{aligned} \tag{2.1}$$

Makinde [16], Chinyoka & Makinde [26] and under stated assumptions the governing equations may be written as given by Makinde et al. [17], Ellahi [27,28]

$$\frac{\partial w}{\partial t} = \frac{1}{r\rho} \left[\frac{\partial}{\partial r} \left(r\mu \frac{\partial w}{\partial r} \right) + \alpha_1 \frac{\partial}{\partial r} \left(r \frac{\partial^2 w}{\partial r \partial t} \right) + 2\beta_3 \frac{\partial}{\partial r} \left(r \left(\frac{\partial w}{\partial r} \right)^3 \right) \right] - \frac{\partial \hat{p}}{\partial z} - \frac{\sigma B_r^2 w}{1 + m^2} \tag{2.2}$$

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\mu}{\rho c_p} \left(\frac{\partial w}{\partial r} \right)^2 + \frac{\alpha_1}{\rho c_p} \frac{\partial^2 w}{\partial r \partial t} \frac{\partial w}{\partial r} + \frac{2\beta_3}{\rho c_p} \left(\frac{\partial w}{\partial r} \right)^4 + \frac{k}{\rho c_p} \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + \frac{Q_s}{\rho c_p} \left(T - T_0 \right) \\ &\quad - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial r} + \frac{D_m \lambda_T}{\rho c_p c_s} \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) \end{aligned} \tag{2.3}$$

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial r} = D_m \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - k_c \left(C - C_0 \right) \tag{2.4}$$

where $w, T, B_0, \hat{p} = -p + \alpha\left(\frac{dw}{dr}\right)^2, \sigma, m, k, q, D_m, \lambda_T, c_p, k_c, T_0, T_w, C_0, C_w$ are fluid velocity, fluid temperature, applied magnetic field strength, modified pressure, electrical conductivity, Hall parameter, thermal conductivity, thermal radiation, molecular diffusivity, thermal diffusivity, specific heat capacity, chemical reaction rate constant, reference temperature, wall temperature, reference concentration and wall concentration.

Introducing the following non-dimensional quantities by Ellahi [28] into (2.2) to (2.5) and the boundary conditions

$$w = w_0\overline{w},\quad t = \frac{\overline{t}R}{w_0},\quad r = R\eta,\quad T = (T_w - T_0)\theta + T_0,\quad C = (C_w - C_0)\chi + C_0$$
$$\Lambda = \frac{2\beta_3 w_0}{\rho R^3}, c = \frac{R}{\rho {w_0}^2}\left(\frac{\partial \overline{p}}{\partial z}\right), R_e = \frac{\rho w_0 R}{\mu_0}\ P_r = \frac{\mu_0 c_p}{k}, M = \frac{\sigma B_0^2 R^2}{\rho w_0}, Q_H = \frac{Q_s R}{\rho w_0 c_p}, \tag{2.6}$$
$$E_c = \frac{w_0^2}{c_p(T_w - T_0)},\ K_R = \frac{K_c R^2}{D_m}, D_u = \frac{D_m \lambda_T (C_w - C_0)}{k c_p (T_w - T_0)}\ , R_p = \frac{16\sigma_* T_0^3}{3\delta_* \rho w_0 c_p R}, \mathrm{S}_c = \frac{w_0 R}{D_m}$$

and using Rosselands approximation

$$q_r = -\frac{4\sigma_*}{3\delta_*}\frac{\partial T^4}{\partial r} \tag{2.7}$$

$\Lambda, M, c,\ P_r, E_c, Q_H,\ \beta_*, D_u\ , R_p, \mathrm{S}_c, K_R, \sigma_*, \delta_*$ denotes third grade parameter, magnetic parameter, pressure drop, Prandtl number, Eckert number, heat source/sink parameter, material constant parameter, Dufour number, radiation parameter, Schmidt number, chemical reaction parameter, Stefan-Boltzmann constant and mean absorption coefficient. For steady flow, the time dependent terms are set to zero and the following are equations were obtained respectively with the boundary conditions

$$\frac{1}{R_e}\frac{d^2 w}{d\eta^2} + \frac{1}{\eta R_e}\frac{dw}{d\eta} + \frac{\Lambda}{\eta}\left(\frac{dw}{d\eta}\right)^3 + 3\Lambda\left(\frac{dw}{d\eta}\right)^2\frac{d^2 w}{d\eta^2} - c - \frac{Me^{\eta} w}{\left(1+m^2\right)} = 0 \tag{2.8}$$

$$\begin{aligned}&\frac{d^2\theta}{d\eta^2} + \frac{R_*}{\eta}\frac{d\theta}{d\eta} + P_r E_c R_*\left(\frac{dw}{d\eta}\right)^2 + \beta_* R_e P_r E_c R_*\left(\frac{dw}{d\eta}\right)^4 + Q_H R_e P_r R_* \theta \\ &+ D_u P_r R_e R_* \frac{d^2\chi}{d\eta^2} + \frac{D_u P_r R_e R_*}{\eta}\frac{d\chi}{d\eta} = 0\end{aligned} \tag{2.9}$$

$$\frac{d^2\chi}{d\eta^2} = S_c w\frac{d\chi}{d\eta} - \frac{1}{\eta}\frac{d\chi}{d\eta} - K_R\chi \tag{2.10}$$

$$\begin{aligned}&\frac{dw}{d\eta} = 0,\ \ \theta(\eta) = 0,\ \ \chi(\eta) = 0 \qquad \text{at} \qquad \eta = 0 \\ &w(\eta) = 0,\ \ \theta(\eta) = 1,\ \ \chi(\eta) = 1 \qquad \text{at} \qquad \eta = 1\end{aligned} \tag{2.11}$$

Equations (2.8), (2.9), (2.10) and (2.11) comprise the boundary value problem to now be solved.

3 Methods

3.1 Galerkin weighted residual methods

Suppose an approximate solution is to be determined for the differential equation of the form

$$L(\phi)+f=0 \tag{3.1}$$

where $\phi(x)$ is an unknown dependent variable, L is a differential operator and $f(x)$ is a known function. Let $\psi(x)=\sum_{i=1}^{N} c_i u_i(x)$ be an approximate solution to (2.8). On substituting $\psi(x)$ into (2.8), it is unlikely that (2.8) is satisfied i.e. $L(\psi)+f\neq 0$ therefore

$$L(\psi)+f=R \tag{3.2}$$

where $R(x)$ is a measure of error called the Residual [23,29]. Multiplying (3.2) by an arbitrary weight function $u(x)$ and integrating over the domain to obtain

$$\int_D u(x)\left[L(\psi)+f\right]dD=\int_D u(x)R(x)dD\neq 0 \tag{3.3}$$

Galerkin Weighted Residual method ensures equation (3.3) vanishes over the solution domain and the weight function is choosing from the basis functions $u(x)=u_i(x) \quad (i=0,...,N)$ hence

$$\langle u,R\rangle=\int_D u(x)R(x)dD=\int_D u_i(x)\left[L(u_0(x)+\sum_{i=1}^{N}c_i u_i(x))+f\right]dD=0 \tag{3.4}$$

These are a set of n-order linear equations to be solved to obtain all the c_i coefficients. The trial functions can be polynomials, trigonometric functions etc. The trial functions are usually chosen in such that the assumed function $\psi(x)$ satisfies the global boundary conditions for $\phi(x)$ though this is not strictly necessary and certainly not always possible [22].

To apply the method to (2.8)-(2.10), we select an approximate solutions of the form $\psi_w(\eta)=a_0+a_1\eta+a_2\eta^2$, $\psi_\theta(\eta)=b_0+b_1\eta+b_2\eta^2$, $\psi_\chi(\eta)=c_0+c_1\eta+c_2\eta^2$ for the velocity, temperature and concentration respectively, which satisfies the boundary conditions (2.11). Applying the boundary conditions on the approximate solution we obtain the following:

$w(\eta)=a_0(1-\eta^2)$, $\theta(\eta)=\eta^2+b_1(\eta-\eta^2)$, $\chi(\eta)=\eta^2+c_1(\eta-\eta^2)$ and $u_1=(1-\eta^2)$, $u_2=(\eta-\eta^2)$, $u_3=(\eta-\eta^2)$ are the weighting functions u_i, where a_0,b_1,c_1 are the coefficients to be determined.

The residue R for (2.7)-(2.9) respectively are given by

$$R_a=1-\frac{4a_0}{R_e}-32\Lambda a_0^3\eta^2+\frac{Me^{\eta}a_0\eta^2}{1+m^2}-\frac{Me^{\eta}a_0}{1+m^2} \tag{3.5}$$

$$R_b = 2(1-R_*) + R_* b_1(\frac{1}{\eta} - 2) - 2b_1 + 4P_r E_c P_* \eta^2 a_0^2 + 16\Lambda P_r E_c R_e P_* \eta^4 a_0^4 - Q_H P_r R_e P_* \eta^2 b_1$$
$$+ Q_H P_r R_e P_* \eta^2 + Q_H P_r R_e P_* \eta b_1 - 4D_u P_r R_e P_* c_1 + 4D_u P_r R_e P_* + \frac{D_u P_r R_e P_* c_1}{\eta} \quad (3.6)$$

$$R_c = 4(1-c_1) + \frac{c_1}{\eta} - 2S_c a_0 c_1 \eta^3 + 2S_c a_0 \eta^3 + 2S_c a_0 c_1 \eta^2 + 2S_c a_0 c_1 \eta - 2S_c a_0 \eta$$
$$-S_c a_0 c_1 - K_R c_1 \eta^2 + K_R \eta^2 + K_R c_1 \eta \quad (3.7)$$

Taking into account of orthogonality of the residues above, we have

$$\langle u_1, R_a \rangle = \int_0^1 \left[(1-\eta^2)(1 - \frac{4a_0}{R_e} - 32\Lambda a_0^3 \eta^2 + \frac{Me^\eta a_0 \eta^2}{1+m^2} - \frac{Me^\eta a_0}{1+m^2}) \right] d\eta = 0$$

$$\langle u_2, R_b \rangle = \int_0^1 \left[\begin{array}{l} (\eta-\eta^2)(2(1-R_*) + R_* b_1(\frac{1}{\eta} - 2) - 2b_1 + 4P_r E_c P_* \eta^2 a_0^2 \\ +16\Lambda P_r E_c R_e P_* \eta^4 a_0^4 - Q_H P_r R_e P_* \eta^2 b_1 + Q_H P_r R_e P_* \eta^2 \\ +Q_H P_r R_e P_* \eta b_1 - 4D_u P_r R_e P_* c_1 + 4D_u P_r R_e P_* + \frac{D_u P_r R_e P_* c_1}{\eta}) \end{array} \right] d\eta = 0$$

$$\langle u_3, R_c \rangle = \int_0^1 \left[\begin{array}{l} (\eta-\eta^2)(4(1-c_1) + \frac{c_1}{\eta} - 2S_c a_0 c_1 \eta^3 + 2S_c a_0 \eta^3 + 2S_c a_0 c_1 \eta^2 \\ +2S_c a_0 c_1 \eta - 2S_c a_0 \eta - S_c a_0 c_1 - K_R c_1 \eta^2 + K_R \eta^2 + K_R c_1 \eta) \end{array} \right] d\eta = 0$$

The symbolic calculation software MAPLE 2016 is used to compute the values of a_0, b_1, c_1 and the approximate solutions.

3.2 Entropy generation

Inherent irreversibility in a pipe flow occurs owing to exchange of energy and momentum within the fluid and the solid boundaries. The entropy generation is owed to heat transfer and the effects of fluid friction. The equation for rate of entropy generation per unit volume [17,21] is given

$$S^m = \frac{k}{T_w^2}\left(\frac{dT}{dr}\right)^2 + \frac{\mu}{T_w}\left(\frac{dw}{dr}\right)^2 + \frac{2\beta_3}{T_w}\left(\frac{dw}{dr}\right)^4 \quad (4.1)$$

where the first term in (4.1) is the irreversibility due to heat transfer, the second and third term are entropy generation due to viscous dissipation. Introducing the dimensionless quantities in (2.6) to (4.1), we have

$$N_s = \frac{r^2 S^m}{k} = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2 + \frac{B_R \eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_* \eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 \quad (4.2)$$

where $\Omega = \dfrac{T_w}{T_w - T_0}, B_R = \dfrac{\mu w_0^{\,2}}{k(T_w - T_0)}, \beta_* = \dfrac{\beta_3 w_0^{\,4}}{kR^2(T_w - T_0)}$ are temperature difference parameter, Brickman number and third grade parameter and

$$N_1 = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2, N_2 = \frac{B_R\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_*\eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 \tag{4.3}$$

where N_1 is irreversibility due to heat transfer and N_2 gives entropy generation due to viscous dissipation. The Bejan number is defined as

$$B_e = {N_1}/{N_s} \tag{4.4}$$

such that $0 \le B_e \le 1$ denoting $B_e = 1$ is the limit at which heat transfer irreversibility dominates, $B_e = 0$ is the limit at which total irreversibility dominates, and $B_e = 1/2$ connotes equal contribution [30].

4 Results and Discussion

In this section, results are presented and discussed. Fig. 1 depicts the influence of magnetic parameter, increasing the magnetic parameter decreases the flow profile of the system owning to the Lorentz force acting in contradiction of the flow. Fig. 2 shows the Hall parameter enhancing the flow profile with increasing Hall values. Increasing the Reynolds number enhances the velocity profile as shown in Fig. 3. In Fig. 4, the thickening effect of the fluid in regard to increasing thirdgrade parameter inhibits the flow field.
Figs. 5-7 portrays the effect of Eckert, Prandtl and Reynolds number on the temperature profile. Considerable increase in the Eckert number slightly increases the temperature profile then increasing the Prandtl number and Reynolds number decreases the temperature field of the system. Since Prandtl number is the ratio of kinematic viscosity to thermal diffusivity so as P_r increases, the kinematic viscosity dominate thermal diffusivity causing the velocity flow field to decrease. The temperature field in Fig. 8 is enhanced with increasing the radiation parameter.

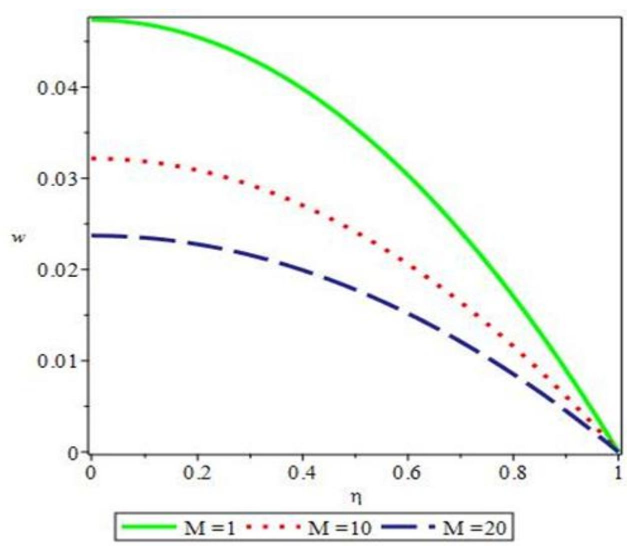

Fig. 1. Effect of varying magnetic parameter (M=1, M=10, M=20) on velocity profile

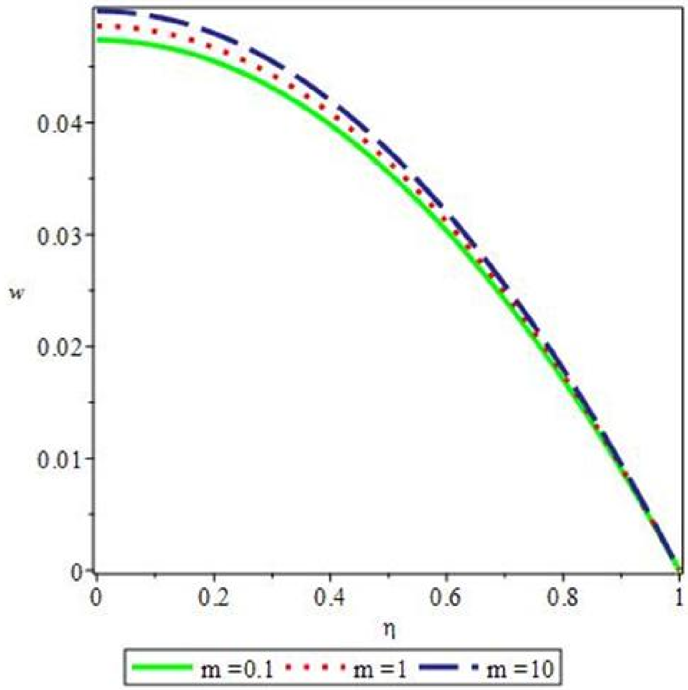

Fig. 2. Effect of varying Hall parameter (m=0.1, m=1, m=10) on velocity profile

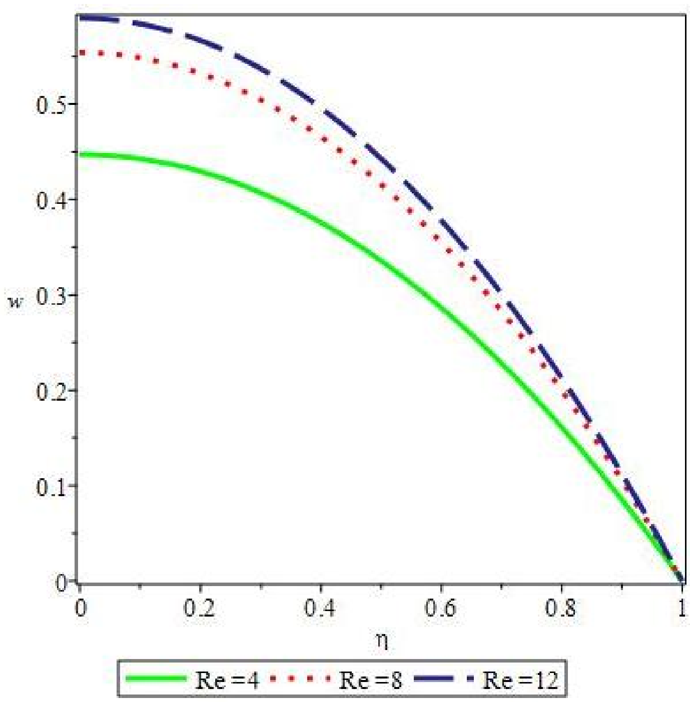

Fig. 3. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on velocity profile

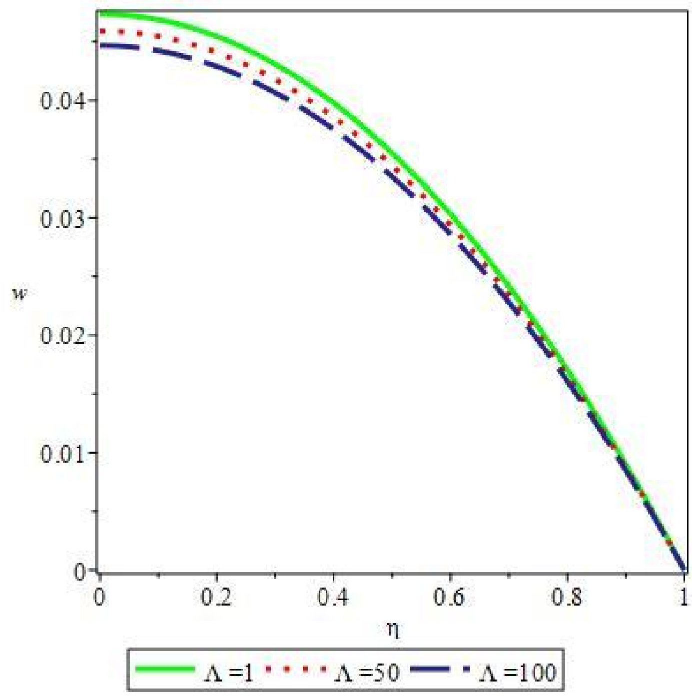

Fig. 4. Effect of varying Thirdgrade parameter (Λ =1, Λ =50, Λ =100) on velocity profile

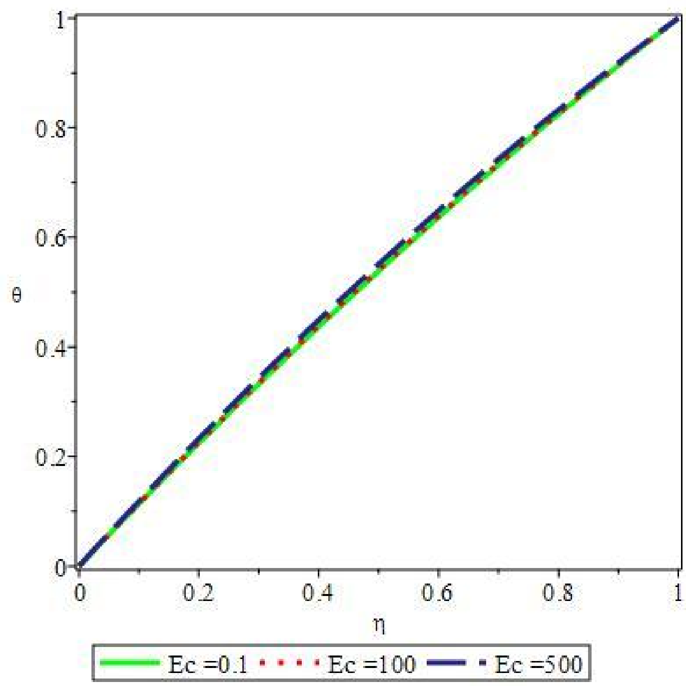

Fig. 5. Effect of varying Eckert number (Ec=0.1, Ec=100, Ec=500) on velocity profile

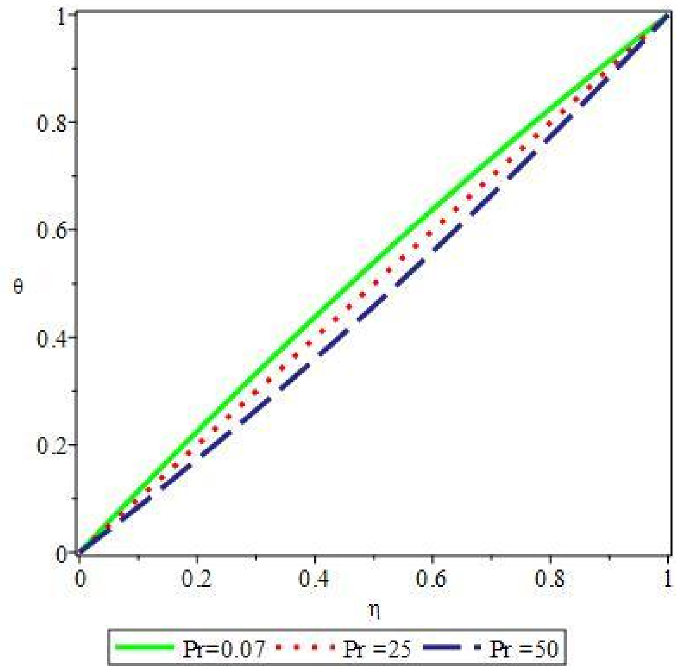

Fig. 6. Effect of varying Prandtl number (Pr=0.07, Pr=25, Pr=50) on temperature profile

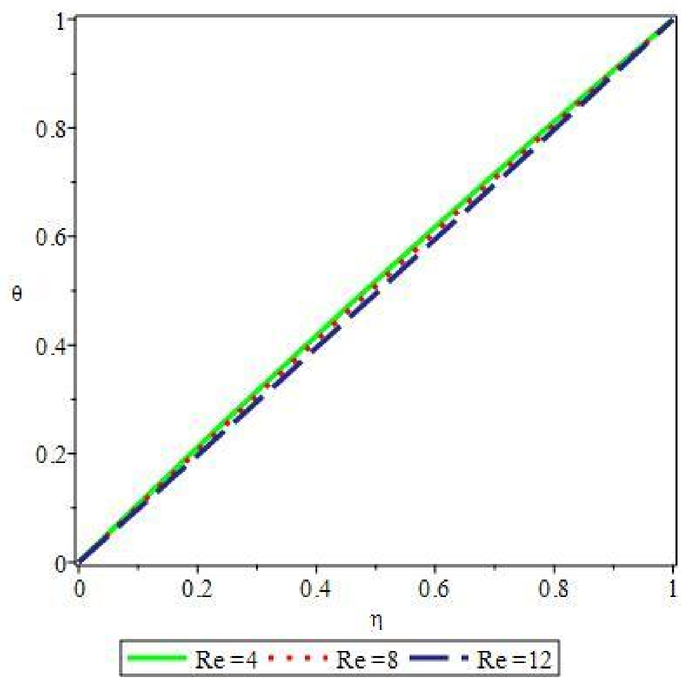

Fig. 7. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on temperature profile

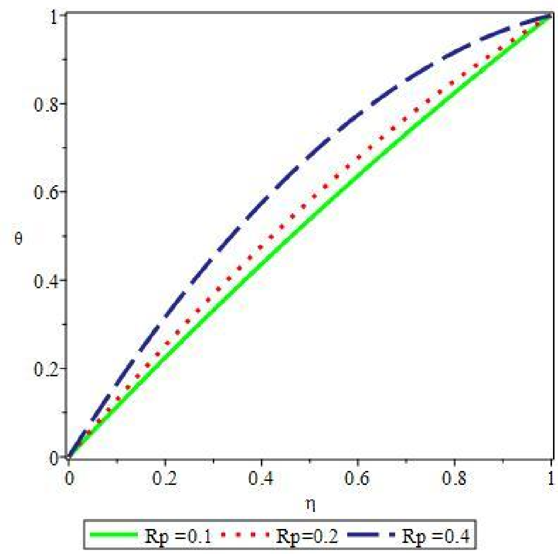

Fig. 8. Effect of varying Radiation parameter (Rp=0.1, Rp=0.2, Rp=0.4) on temperature profile

Figs. 9-10 depicts the influence of Dufour and Schmidt numbers on the concentration profile. Increasing the Dufour number increases the concentration field while the concentration profile decreases with increasing values of Schmidt number. This shows that heavier diffusing species have a greater retarding effect on the concentration distribution. The entropy generation profile is portrayed in Figs. 11-14 with influences of Reynolds, Prandtl, Eckert numbers and radiation parameter. Increasing the Reynolds number enhances the entropy generation while increasing Eckert number inhibits entropy generation. Increasing the Prandtl number decreases the entropy generation firstly around the pipe centreline then it enhances entropy rapidly towards the pipe wall while increasing the radiation parameter enhances the entropy generation around the centreline firstly then it inhibits it rapidly towards the pipe wall.

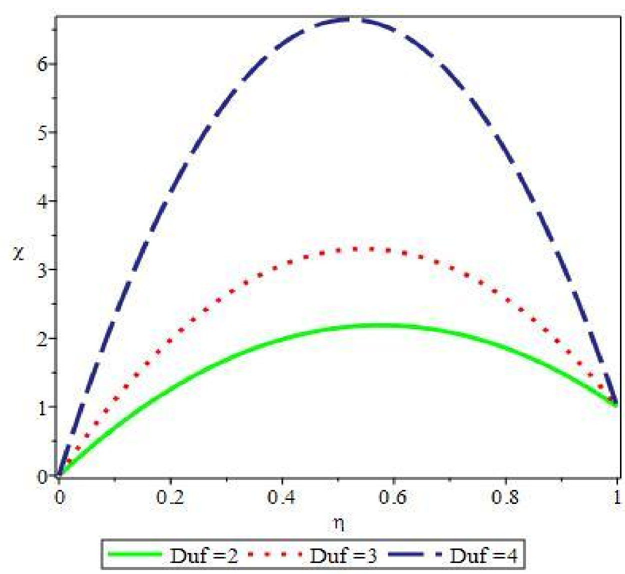

Fig. 9. Effect of varying Dufour number (Duf=2, Duf=3, Duf=4) on concentration profile

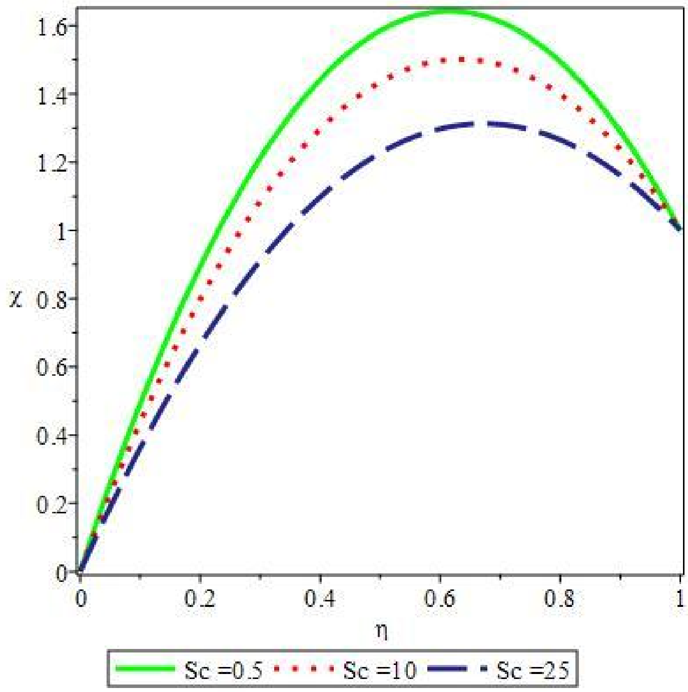

Fig. 10. Effect of varying Schmidt number (Sc=0.5, Sc=10, Sc=25) on concentration profile

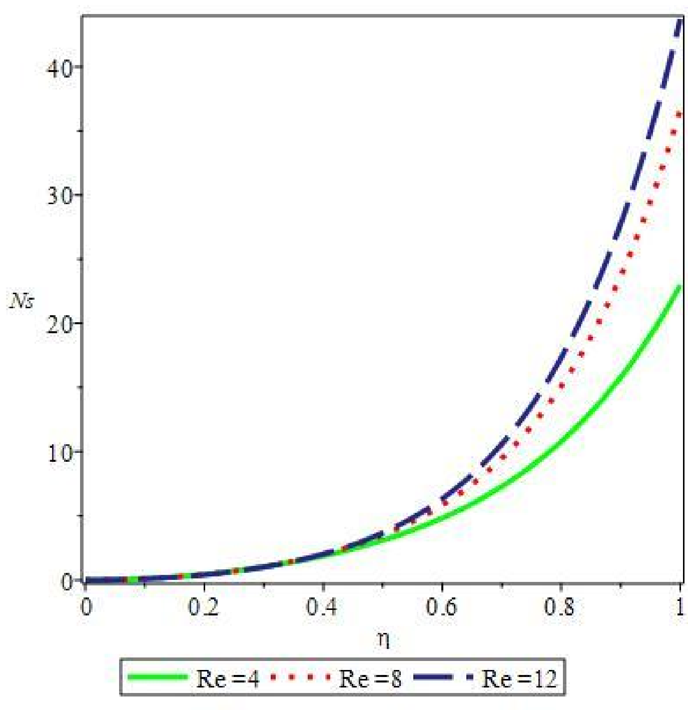

Fig. 11. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on entropy generation profile

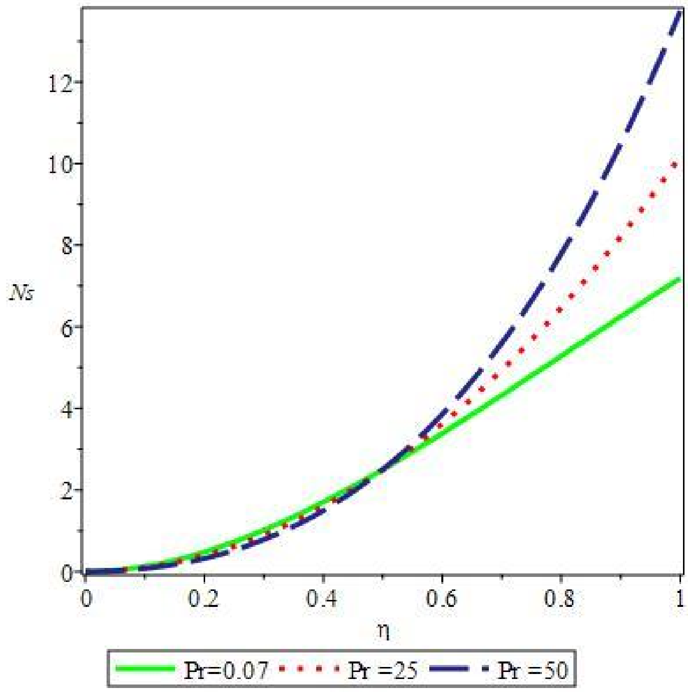

Fig. 12. Effect of varying Prandtl number (Pr=0.07, Pr=25, Pr=50) on entropy generation profile

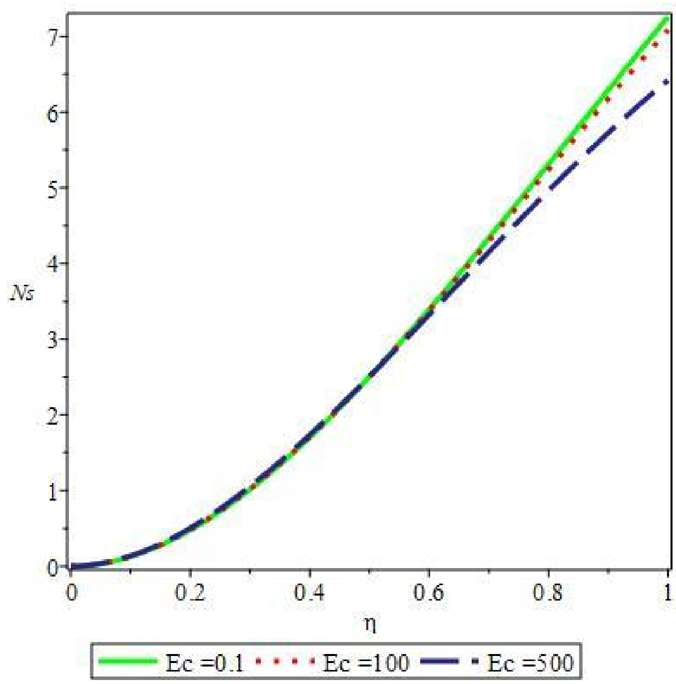

Fig. 13. Effect of varying Eckert number (Ec=0.1, Ec=100, Ec=500) on entropy generation profile

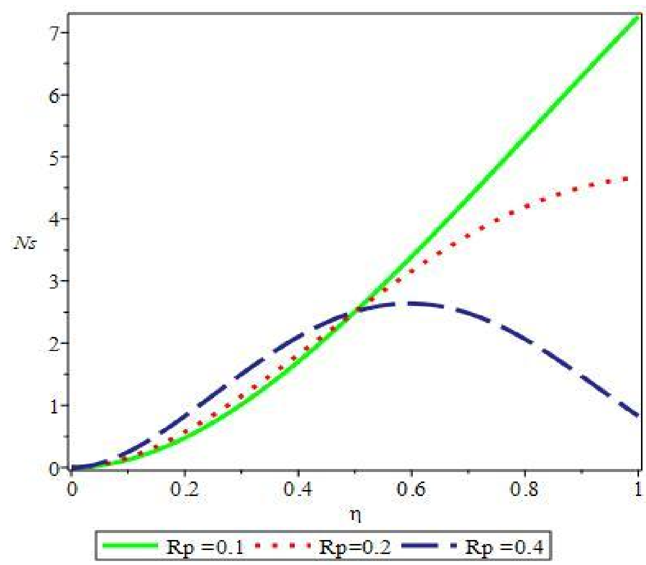

Fig. 14. Effect of varying Radiation parameter (Rp=0.1, Rp=0.2, Rp=0.4) on entropy generation profile

Figs. 15-22 presents the influence of Hall parameter, magnetic parameter, Prandtl number, Eckert number Reynolds number, thirdgrade parameter, Dufour number and reaction parameter on Bejan number. Increasing the Hall parameter, Eckert number and Reynolds number inhibits the Bejan number and the irreversibility due to heat transfer dominates over total irreversibility from the pipe centreline to pipe wall except for Reynolds number where irreversibility due to total dominates gradually towards the pipe wall. On increasing the magnetic parameter, thirdgrade parameter, Dufour number and reaction parameter enhances the Bejan number and the irreversibility due to heat transfer dominates over total irreversibility. Increasing the Prandtl number firstly inhibits the Bejan number around the pipe centreline then it enhances Bejan number towards the wall of the pipe and the flow is dominated by heat transfer irreversibility.

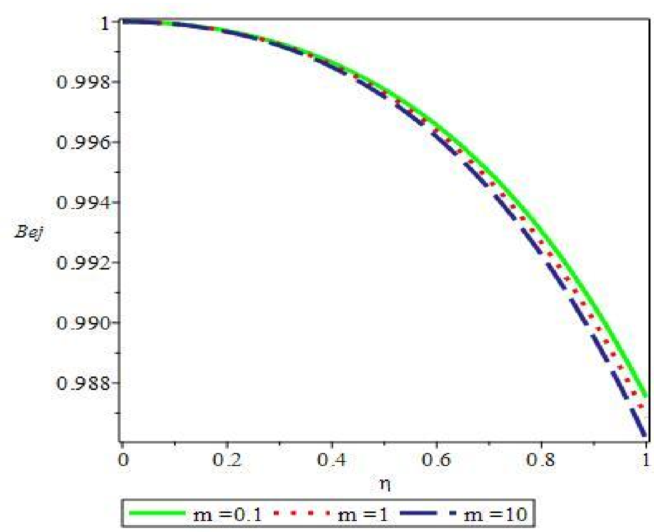

Fig. 15. Effect of varying Hall parameter (m=0.1, m=1, m=10) on Bejan number

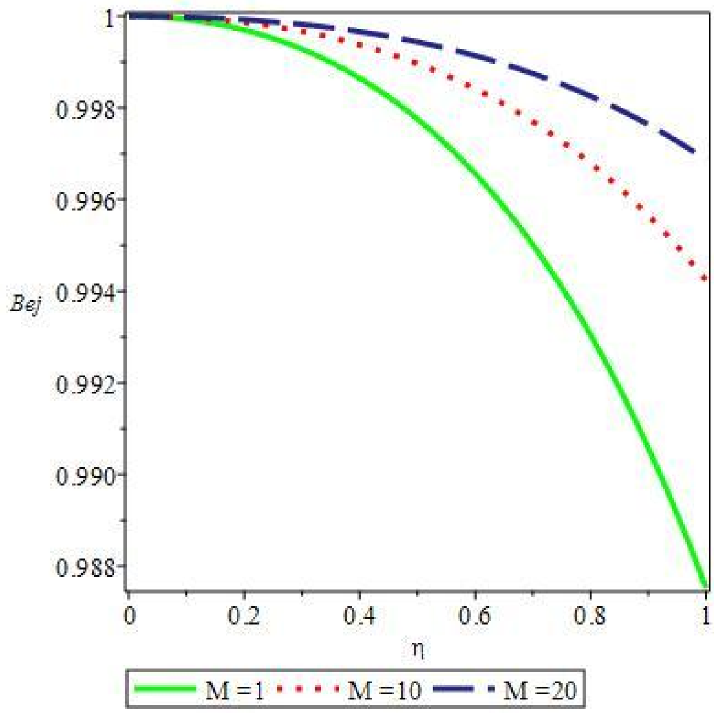

Fig. 16. Effect of varying Magnetic parameter (M=1, M=10, M=20) on Bejan number

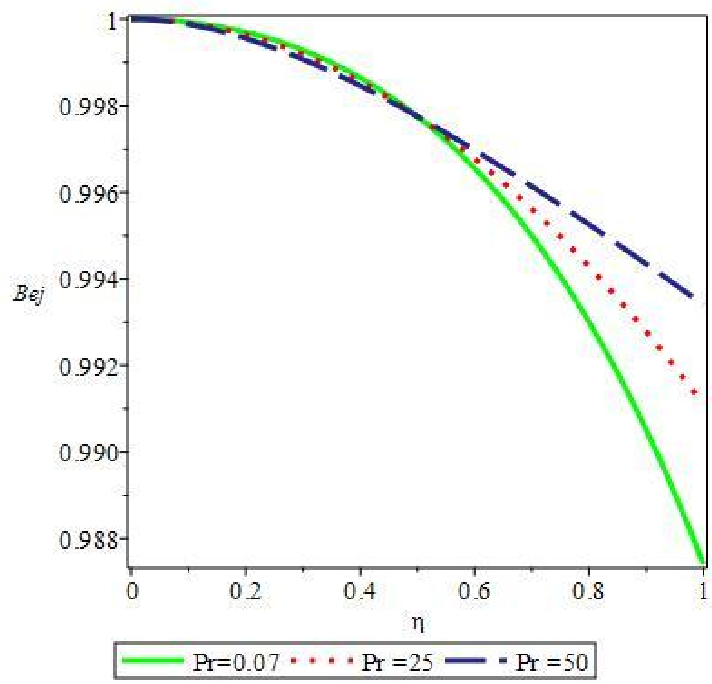

Fig. 17. Effect of varying Prandtl number (Pr=0.07, Pr=25, Pr=50) on Bejan number

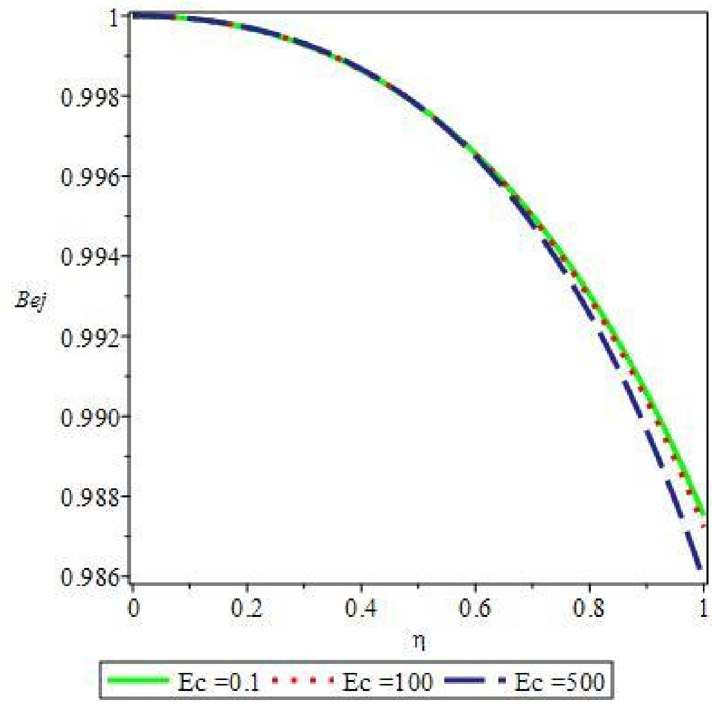

Fig. 18. Effect of varying Eckert number (Ec=0.1, Ec=100, Ec=500) on Bejan number

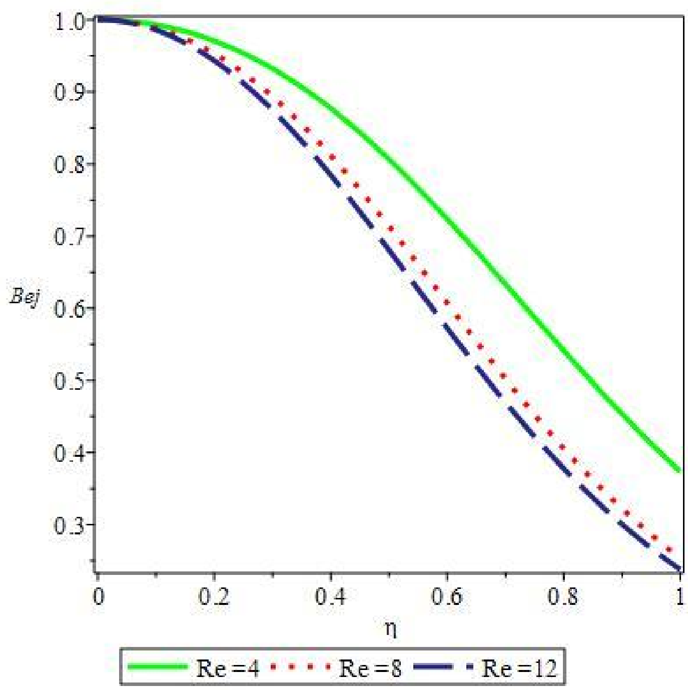

Fig. 19. Effect of varying Reynolds number (Re=4, Re=8, Re=12) on Bejan number

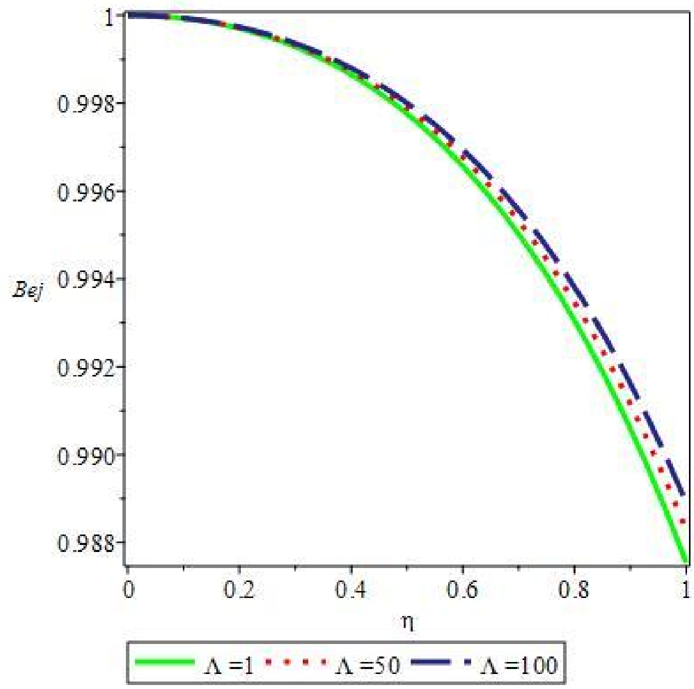

Fig. 20. Effect of varying Thirdgrade parameter (Λ =1, Λ =50, Λ =12) on Bejan number

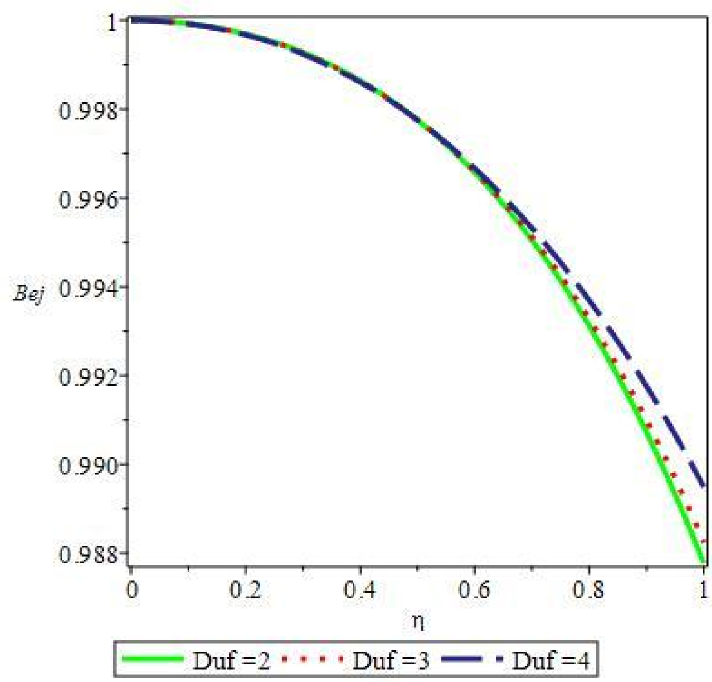

Fig. 21. Effect of varying Dufour number (Duf=2, Duf=3, Duf=4) on Bejan number

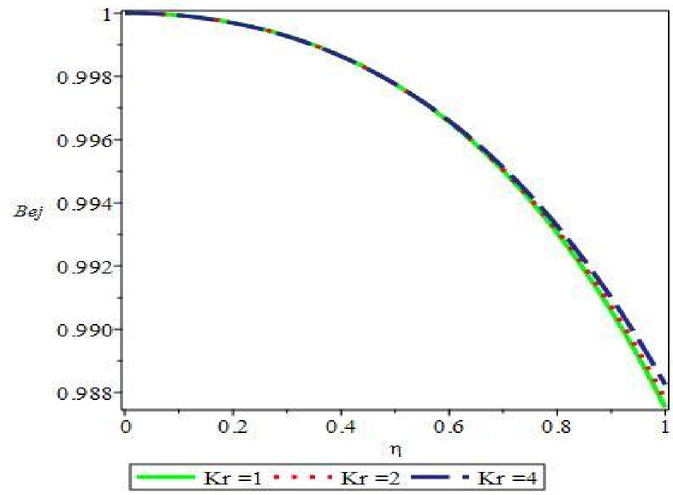

Fig. 22. Effect of varying reaction parameter (Kr=1, Kr=2, Kr=4) on Bejan number

5 Conclusion

In this numerical investigation, the entropy generation rate of steady reactive magnetohydrodynamic third grade fluid flow in a circular pipe is presented using the Galerkin method. Numerical expression for the velocity, temperature and concentration was obtained which were used to compute the entropy generation number. Special emphasis has been focused on the variations of pertinent parameter of physical significance on the entropy generation rate and Bejan. The main findings of the present analysis are:

- The velocity is enhanced for increasing values of m, Re and inhibited for M, Λ
- The temperature is enhanced for values of Ec, Rp and inhibited for $\mathrm{P}_r, \mathrm{Re}$ and Du
- The concentration is enhanced values of Du, K_R and inhibited for Sc and Re
- Re, K_R and Du have enhancing effects on the entropy generation rate.
- M, Du, K_R and Λ enhances the entropy generation rate while it is inhibited for Re and Ec.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Veera Krishna M, Gangadhar Reddy M. MHD free convective boundary layer flow through porous medium past a moving vertical plate with heat source and chemical reaction. Materials Today: Proceedings. 2018;5:91–98.
Available:https://doi.org/10.1016/j.matpr.2017.11.058

[2] Veera Krishna M, Subba Reddy G. MHD forced convective flow of Non-Newtonian fluid through stumpy permeable porous medium. Materials Today: Proceedings. 2018;5:175–183.
Available:https://doi.org/10.1016/j.matpr.2017.11.069

[3] Veera Krishna M, Kamboji Jyothi. Hall effects on MHD rotating flow of a visco-elastic fluid through a porous medium over an infinite oscillating porous plate with heat source and chemical reaction. Materials Today: Proceedings. 2018;5:367–380.
Available:https://doi.org/10.1016/j.matpr.2017.11.094

[4] Reddy BSK, Veera Krishna M, Rao KVSN, Bhuvana Vijaya R. HAM solutions on MHD flow of nano-fluid through saturated porous medium with Hall effects. Materials Today: Proceedings. 2018;5:120–131.
Available:https://doi.org/10.1016/j.matpr.2017.11.062

[5] Veera Krishna M, Swarnalathamma BV. Convective heat and mass transfer on MHD peristaltic flow of Williamson fluid with the effect of inclined magnetic field. AIP Conference Proceedings. 2016;1728:020461.
DOI: 10.1063/1.4946512

[6] Swarnalathamma BV, Veera Krishna M. Peristaltic hemodynamic flow of couple stress fluid through a porous medium under the influence of magnetic field with slip effect. AIP Conference Proceedings. 2016;1728:020603.
DOI: 10.1063/1.4946654

[7] Veera Krishna M, Gangadhar Reddy M. MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating porous plate with chemical reaction. IOP Conf. Series: Materials Science and Engineering. 2016;149:012217.
DOI: 10.1088/1757-899X/149/1/012217

[8] Veera Krishna M, Subba Reddy G. Unsteady MHD convective flow of second grade fluid through a porous medium in a rotating parallel plate channel with temperature dependent source. IOP Conf. Series: Materials Science and Engineering. 2016;149:012216.
DOI: 10.1088/1757-899X/149/1/012216

[9] Veera Krishna M, Swarnalathamma BV, Prakash J. Heat and mass transfer on unsteady MHD oscillatory flow of blood through porous arteriole. Applications of Fluid Dynamics, Lecture Notes in Mechanical Engineering. 2018;22:207-224.
DOI: 10.1007/978-981-10-5329-0_14

[10] Veera Krishna M, Subba Reddy G, Chamkha AJ. Hall effects on unsteady MHD oscillatory free convective flow of second grade fluid through porous medium between two vertical plates. Physics of Fluids. 2018;30:023106.
DOI: 10.1063/1.5010863

[11] Veera Krishna M, Chamkha AJ. Hall effects on unsteady MHD flow of second grade fluid through porous medium with ramped wall temperature and ramped surface concentration. Physics of Fluids. 2018;30:053101.
DOI: https://doi.org/10.1063/1.5025542

[12] Taza G, Rehan A, Saeed I, Mohammed A. MHD thin film flows of a third grade fluid on a vertical belt with slip boundary conditions. Journal of Applied Mathematics; 2013.
DOI: dx.doi.org/10.1155/2013/707286

[13] Veera Krishna M, Gangadhara Reddy M, Chamkha AJ. Heat and mass transfer on MHD free convective flow over an infinite non-conducting vertical flat porous plate. Int. Jour. of Fluid Mech. Res. 2018c;45(5):1-25.
DOI: 10.1615/InterJFluidMechRes.2018025004

[14] Veera Krishna M, Jyothi K. Heat and mass transfer on MHD rotating flow of a visco-elastic fluid through porous medium with time dependent oscillatory permeability. J. Anal. 2018;25(2):1-19.
Available:https://doi.org/10.1007/s41478-018-0099-0

[15] Taza G, Saed I, Rehan A, Ilyas K, Sharidan S. Thin flow in MHD third grade fluid on a vertical belt with temperature dependent viscosity. PLoS. 2014;9(6).
Available:https://doi.org/10.1371/journal.pone.0097552

[16] Makinde OD. On steady flow of a reactive variable fluid in a cylindrical pipe with an isothermal wall. International Journal of Numerical Methods for Heat & Fluid Flow. 2007;17(2):187-194.

[17] Makinde OD, Chinyoka T, Eegunjobi AS. Numerical investigation of Entropy generation in an unsteady flow through a porous pipe with suction. Int. J. Exergy. 2013;12(3):279-299.

[18] Ajadi SO. A note on the thermal stability of a reactive non-Newtonian flow in a cylindrical pipe. International Communications in Heat and Mass Transfer. 2009;36:63-68.

[19] Aiyesimi YM, Okedayo GT, Lawal OW. Analysis of magnetohydro-dynamics flow of a dusty viscoelastic fluid through a horizontal circular channel. Academia Journal of Scientific Research. 2013;1(3):056-062.

[20] Eeegunjobi AS, Makinde OD. Combined effect of Buoyancy force and Navier slip on Entropy generation in a vertical porous channel. Entropy. 2012;14:1028-1044.

[21] Chinyoka T, Makinde OD. Analysis of Entropy generation in an unsteady porous channel flow with Navier slip and convective cooling. Entropy. 2013;15:2081-2099.

[22] Finlayson BA. The method of weighted residuals and variational principles. New York and London: Academic Press; 1972.

[23] Jain MK. Numerical solution of differential equations. 2nd Ed. New Dehli: Wiley Eastern Ltd.; 1984.

[24] Barletta A, Lazzari S, Magyari E, Pop I. Mixed convection with heating effects in a vertical porous annulus with a varying magnetic field. International Journal of Heat and Mass Transfer. 2008;51:5777-5784.

[25] Rajagopal KR. Viscometric flows of third grade fluids. Mechanics Research Communications. 1980;7(1):21-25.

[26] Chinyoka T, Makinde OD. Computational dynamics of unsteady flow of a variable viscosity reactive fluid in a porous pipe. Mechanics Research Communications. 2010;37:347-353.

[27] Ellahi R, Riaz Arshad. Analytical solutions for MHD flow in a third grade with variable viscosity. Mathematical and Computer Modelling. 2010;52:1783-1793.

[28] Ellahi R. The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe: Analytical solutions. Applied Mathematical Modelling. 2013;37:1451-1467.

[29] Baluch MH, Moshen MF, Ali AI. Method of weighted residuals as applied to nonlinear differential equations. Appl. Math. Modelling. 1983;7:362-365.

[30] Bejan A. Entropy generation minimization. New York: CRC Press; 1996.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle3.com/review-history/43604