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Modelling Dynamic Micro and Macro Panel Data with Autocorrelated Error Terms

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Authors' contributions

This work was carried out in collaboration among all authors. Author KTU designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors APO and IFO managed the analyses of the study. Author TMA managed the literature searches. All authors read and approved the final manuscript.

Original Research Article

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Abstract

Aims: The aim of this study is to determine the best estimator for estimating dynamic panel data model with serially uncorrelated disturbances and exogenous regressors.
Methodology: In this study, properties of some Dynamic Panel Data estimators are investigated. These are Ordinary Least Squares (OLS), the Anderson-Hsiao(AH(d), Arellano-Bond Generalized Method of Moment (ABGMM) one-step, Blundell- Bond System (BBS) one-step, M- estimator, MM estimators and proposed estimator, Modified Anderson-Hsiao with Arellano-Bond(MAHAB) estimator in the presence of autocorrelation. Also, this new estimator was proposed by modifying the existing estimators.
Results: Monte-Carlo simulations were carried out at varying sample size (n) ranges from 10-200 and time period (T) ranges from 5-20 when autocorrelation (δ) is fixed at 0.3, 0.5 and 0.7. The estimators considered performed well except OLS and BBS for all time periods.

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Conclusion: AH estimator performed relatively well when the time period is small while ABGMM estimator outperformed all other estimators when sample size (n) is large for all the time periods considered. ABGMM shows the largest improvement as sample size (n) and time periods (T) increase. The MAHAB estimator outperformed all other estimators in small and large sample size irrespective of time period in the presence of autocorrelation.

1 Introduction

Panel data set is a cross-section or group of entities that are surveyed periodically over a given time span. These data consist of repeated observations on some subjects at different occasions,   generated by pooling time-series observations across a variety of cross-sectional units. The units may be individuals, households, firms, regions or countries. Analysis on panel is classified as Micro panels (involve a number of households or individuals) and Macro panels (involve a number of countries). There are several benefits panel data over conventional cross-sectional and time-series data as described by [1,2]. Among the benefits are accurate inference of model parameters is obtained when dealing with panel data. Also, it has more degrees of freedom and sample variability than cross-sectional data, time-series data for T=1 and N=1 respectively, hence improving the efficiency of econometric estimates. [3] extended standard error components model to take into account serial correlation.

Heteroscedastic as well as serially correlated disturbances in one way error component was examined in a panel data regression model both in static and dynamic [2]. The problems of autocorrelation due to the presence of lagged dependent variable among the regressors and individual effects characterizing the heterogeneity among the individuals leads to certain issues which are dealt with by different estimation techniques. The estimation of fixed effects dynamic panel data models has been one of the major challenges in Econometrics in the last three decades.

A number of techniques for modeling dynamic panel data have been proposed and compared with Instrumental Variable (IV) and Generalized Method of Moments (GMM) estimators [4]. Therefore, this study will examine the performance of different estimators from small samples to large samples with different time dimension. [5] favorably compared the AH estimator against various GMM estimators.

Arellano M and Bond S [6] made a deduction on the Anderson-Hsiao estimator against different Generalized Method of Moments (GMM) estimators and inferred that the Generalized Method of Moments (GMM) procedures produce substantial efficiency gains. Their results also showed that GMM1 performed better than GMM2 in both their bias and root mean square error.

Judson R and Owen L [7] considered four estimators: an instrumental variables estimator proposed by [8], two Generalized Method of Moments estimator proposed by [6] and a corrected Least Square Dummy Variables estimator (LSDVC) derived by [5]. Their results confirmed some research work conclusions about OLS and LSDV estimators: (1) in both cases, the bias of γ are more severe than that of β . (2) OLS showed biased estimates even for large T and (3) the bias of the LSDV estimator increases with γ and decrease with T. Their result also showed that the bias of LSDV estimate is not unsubstantial when T =20, but when T increase to 30, the average bias becomes smaller although the LSDV does not become efficient. However, all the estimators' performs better with a larger N and T, and the one-step GMM also performs better than the two-step GMM estimator. Their result also showed that LSDV performs just as well as the viable alternatives when T=30, GMM is the best when T ≤ 10 and GMM or AH may be chosen when T=20.

Nerlove M [9] compared the Least Trimmed Squares estimators (LTS), M-estimator, Yohai MM-estimator, S-estimator and Ordinary Least Squares. The simulation results showed that the S-estimator, M-estimator methods perform better than Least Trimmed Squares and MM-estimator methods. The result also showed that the S-estimator has a reasonable efficiency, with the influence of high leverage outliers, and demonstrate high

breakdown. For 10% breakdown S-estimator increases its efficiency. MM-estimation performed the best overall against a comprehensive set of outlier conditions. However, his results also showed that when the percentages of outliers are increased, the performances of the estimators were reduced.

Alma Ö G [10] compared the Anderson-Hsiao estimator using lagged levels as instrument (AH (l)), Anderson-Hsiao using lagged differences as instrument (AH (d)), Arellano-Bond GMM estimator (first and second step), Blundell-Bond GMM estimator (first and second step). Their simulation result revealed that AH(l) and AH(d) performed reasonably when the time period is small and when time period is moderate while first-step Arellano-Bond GMM estimator performs better than all other estimators when the time period is large. Meanwhile, the first-step Blundell-Bond system GMM estimators do not perform well when the panel sample size is large.

In this study it was found that MAHAB estimator is the appropriate choice in a dynamic panel data model with serially correlated disturbances and exogenous regressors. The rest of the paper is organized as follows: section 1 gives the brief description of SEM models and the interpretation of the terms. Section 2 describes materials and methods, in section 3 and 4 simulation study and results of the simulation study respectively. Section 5 concludes the paper.

2 Materials and Methods

This work considers one-way error component model with presence of serial correlation in a random effects. The different degrees of autocorrelation were introduced via random effects one-way error component model and the coefficient of the serial correlation is taken to be mild, moderate and high. This is in line with the works of [4,11,7] to mention but few. Most of the previous works done on Dynamic panel data focused on the absence or no serial correlation of the disturbance term.

2.1 Frameworks of some estimators of dynamic panel data models considered

Consider: Ordinary Least Square estimators,

$$
\begin{aligned}
y &= (y_{11}, \Lambda, y_{N1}, \Lambda, y_{iT}, \Lambda, y_{NT}), \\
y_{-1} &= (y_{10}, \Lambda, y_{N0}, \Lambda, y_{i,T-1}, \Lambda, y_{N,T-1}), \\
x &= (x_{10}, \Lambda, x_{N0}, \Lambda, x_{i,T-1}, \Lambda, x_{N,T-1})
\end{aligned}
\tag{1}
$$

Also, let $W = [y_{-1}x]$. Then the OLS estimator of the parameter vector $(\alpha\beta)' = \gamma$ is given by

$$
\gamma = (W'W)^{-1}W'y .
\tag{2}
$$

The standard errors under homoscedasticity are obtained from $\text{var}(\gamma) = s^2(W'W)^{-1}$, with $s^2 = e'e / (NT-2)$, where $e = (y - W\gamma)$. The general heteroskedasticity consistent standard errors are obtained from $(W'W)^{-1}W'diag(e'e)W(W'W)^{-1}$. Since $Cov(y_{i,t-1}\mu_i \neq 0)$ OLS estimator is biased. It is also inconsistent in direction of both N and T.

Anderson TW and Hsiao C [8] proposed an instrumental Variable (IV) estimator that is consistent for fixed T and N tends to infinity. Anderson and Hsiao (IV) estimator was applied to the model in first differenced form

$$
y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2})(x'_{it} + x'_{i,t-1})\beta + v_{it} + v_{i,t-1}
\tag{3}
$$

which cancelled the individual fixed effects assumed to possibly correlate with the exogenous variables $(E(x'_{it} - \mu) \neq 0)$ and it resulted in the "loss" of one cross-section from the actual estimation.

The use of level Instruments y_{t-2} was also suggested, or the lagged difference $y_{i,t-2} - y_{i,t-3}$ as an instrument for the differenced lagged endogenous regressor $y_{i,t-1} - y_{i,t-2}$.

Anderson-Hsiao (AH) estimator,

$$\hat{\gamma}^{AH} = (XPX)^{-1} X'Py \quad \text{Where} \quad P = Z(Z'Z)^{-1} Z \tag{4}$$

The symbol l or d indicates the use of levels or differences as instrument $\left(\hat{\gamma}^{AH(l)}, \hat{\gamma}^{AH(d)} \right)$

AB estimator is similar to the one suggested by AH but exploits additional moment restrictions, which enlarges the set of instruments. The dynamic equation to be estimated in levels is $y_{it} = \delta y_{i,t-1} + x'_{it} \beta + \mu_i + v_{it}$ where the individual effect μ_i is eliminated by differencing

$$y_{it} - y_{i,t-1} = \delta \left(y_{i,t-1} - y_{i,t-2} \right) + \left(x'_{it} - x'_{i,t-1} \right) \beta + v_{it} - v_{i,t-1}. \tag{5}$$

The instruments available were looking into for instrumenting the difference equation for each year. For t=3 the equation to be estimated is

$$y_{i3} - y_{i2} = \delta \left(y_{i2} - y_{i1} \right) + \left(x'_{i3} - x'_{i2} \right) \beta + v_{i3} - v_{i2} \tag{6}$$

Where the instruments (again assuming x being at least predetermined) y_{i1}, x'_{i2} and x'_{i1} are available. For t=4 the equation is

$$y_{i4} - y_{i5} = \delta \left(y_{i3} - y_{i2} \right) + \left(x'_{i4} - x'i_3 \right) \beta + v_{i4} - vi_3. \tag{7}$$

The instruments $y_{i,1}, yi_{,2}, x'_{i1}, x'_{i2}$ and x'_{i3} are available.

Arellano-Bond (AB) estimator,

$$\hat{\gamma}^{ABGMM} = \left(XW\hat{V}W'X \right)^{-1} X'W\hat{V}^{-1}W'Y. \tag{8}$$

Where the one-step GMM estimator makes use of a covariance matrix taking autocorrelation into account.

$$V = W'GW = \sum_{i=1}^{N} W'G_T W_i \tag{9}$$

The two-step GMM estimator makes use of the residuals of the first-step estimation to estimate the covariance matrix as suggested by [12]:

$$\hat{V} = \sum_{i=1}^{N} W'_i F_T \hat{V}_i \hat{V}'_i F_T W_i. \tag{10}$$

The BB System GMM: When the instruments are weak the GMM estimator suggested by AB is known to be rather inefficient because of the use of the information contained only in differences. The BB suggests making use of additional level information beside the differences to make it an efficient estimator.

Blundell-Bond (BB) estimator,

$$\hat{\gamma}^{GMM-SSY} = \left(XW\hat{V}^{-1}W'X\right)^{-1} X'W\hat{V}^{-1}Wy. \tag{11}$$

The first step GMM estimator makes use of a covariance matrix taking this autocorrelation into account, enlarged for the level equations while the second step GMM estimator uses residuals of the first step estimation to estimate the matrix as suggested by [12].

M-estimators were proposed by [13]. M-estimation for regression is a relatively straightforward extension of M-estimation for location and scale. It represents one of the first attempts at a compromise between the efficiency of the Least Squares estimator and a resistance estimator - Least Absolute Value (LAV) estimators. Newton-Raphson and Iteratively Reweighted Least Squares (IRLS) are the two methods to the M-estimates nonlinear normal equations.

$$\sum_{i=1}^{n} \Psi\left(\frac{y_i - x'\beta_1}{s}\right)x_i = 0 \tag{12}$$

IRLS express the normal equations as

$$X'WX\hat{\beta} = X'Wy \tag{13}$$

where W is an n x n diagonal matrix of weights

The initial vector of parameter estimates $\hat{\beta}_0$ are typically obtained from OLS,

M estimator:

$$\hat{\beta}_1 = \left(X'WX\right)^{-1} X'Wy \tag{14}$$

MM estimation was introduced by [11] and it combines a high breakdown point with a good efficiency (approximately 95%) relative to the Ordinary Least Squares estimator under the Gauss-Markov assumptions. The MM refers to the fact that Multiple M-estimation procedure is used to calculate the final estimate. It has also become most common robust regression technique for linear regression.

Modified Anderson-Hsiao with Arellano-Bond(MAHAB) estimator: This is the proposed estimator by modification of Anderson-Hsiao and Arellano-Bond estimator given below. For simplicity, considering $\beta = 0$

$$y_{it} = \delta y_{i,t-1} + \mu_i + \upsilon_{it} \tag{15}$$

$$y_{i,t-1} = \delta y_{it-2} + \mu_i + v_{it-1} \tag{16}$$

Subtracting equation (16) from equation (15)

$$y_{it} - y_{it-1} = \delta\left(y_{it-1} - y_{it-2}\right) + \mu_i - \mu_i + v_{it} - v_{it-1} \tag{17}$$

Equation (17) compactly written as follows

$$\Delta y_{i,t} = \delta\Delta y_{i,t-1} + \Delta \upsilon_{it} \tag{18}$$

Firstly, equation (18) was differentiated to eliminate the individual effects. The periods (T) for which there exists valid instruments using logic are $y_{i1,} y_{i2,} \ldots, y_{i,T-2}$

The instrumented equation then becomes

$$W'Fy = W'Fxy + W'Fv \qquad (19)$$

Where
$$X_i = \begin{bmatrix} y_{i2} - y_{i,1} & x'_{i3} - x'_{i2} \\ y_{i3} - y_{i,2} & x'_{i4} - x'_{i3} \\ \vdots & \vdots \\ y_{i,T-1} - y_{i,T-2} & x'_{i,T} - x'_{i,T-1} \end{bmatrix}$$

$$X = [y_{-1}, X] \hat{\gamma} = (\delta, \beta') W = [W'_1, W'_2, \ldots, W'_N]$$

For each individual, define W_i as follows

$$W_i = \begin{bmatrix} [y_{i1}] & \cdots & \cdots & 0 \\ 0 & [y_{i1}, y_{i2}] & & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & [y_{i,1}, \ldots, y_{i,T-2}] \end{bmatrix} \text{ is } (T-2) \times (T-2)(T-1)/2 \text{ matrix.}$$

$$y_i = \begin{bmatrix} y_{i,4} - y_{i,3} \\ y_{i5} - y_{i,4} \\ \vdots \\ y_{iT} - y_{i,T-1} \end{bmatrix}$$

$$V = W'GW = \sum_{i=1}^{N} W_i G_T W_I$$

Where
$$G = (I_N \otimes G_T) = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

premultiplying the matrix F results in transforming the original observations into differences. Because $Var(Fu) = F\sigma^2 F'$, the covariance matrix $V = FF'$ is used as a first step approximation to the covariance matrix.

The two-step GMM estimator uses the residuals of the first-step estimation to estimate the covariance matrix as suggested by [14, 15,16]:

$$\hat{V} = \sum_{i=1}^{N} W'_i F_T v'_i v'_i F_T W_i \qquad (20)$$

Finally, the resulting estimator is

$$\hat{\gamma} = (XW\hat{V}WX)^{-1} X'WV^{-1}Wy \qquad (21)$$

Integrating the blinded model/ estimation with the instrument form

$$\hat{\gamma} = (X'PX)^{-1} XPy, \text{ where } P = Z(Z'Z)^{-1}Z \qquad (22)$$

Premultiply to get and replacing in the formula for A-H estimator

$$\hat{\gamma} = \left(X\!PWV^{-1}WPX \right)^{-1} X\!'PWV^{-1}WPy \tag{23}$$

3 Results and Discussion

3.1 Simulation study

Monte-Carlo experiments were carried out to compare the behaviour of different estimators under different circumstances. The parameter δ and β were compactly given as $\gamma = (\delta, \beta)^1$ for the value of $\beta = 1$. the parameters that are varied in the simulation are autoregressive coefficient (δ, λ) and the autocorrelation coefficient (ρ, θ). The values of $\delta = (0.3, 0.5, 0.7)$, $\lambda = (0.3, 0.5, 0.7)$, $\rho = (0.2, 0.5, 0.9)$ and $\theta = (0.2, 0.5, 0.9)$ for combination of the sample size(N=10, 20, 50,100) and Time period (T=5, 10, 15, 20) with 1000 replications were varied in the study. The assessments of the various estimators considered in this work were based on the RMSE of parameter estimates.

The data generating process follows [12,13]

$$y_{it} = \delta y_{i,t-1} + X^1{}_{it}\beta + \mu_i + v_{it} \tag{24}$$

$$x_{it} = \lambda x_{i,t-1} + \varepsilon_t \tag{25}$$

Where $xit \approx u(-0.5, 0.5)$

For the random effects specification, we generate $u_{it} = \mu_i + v_{it}$ where $\mu_i \sim$ N (0, 1) and error term v_{it} is generated by

$$\text{AR (1):} \quad v_{it} = \rho v_{i,t-1} + w_{it} \tag{26}$$

Or by the MA (1) process

$$v_{it} = w_{it} + \theta w_{i,t-1} \tag{27}$$

3.2 Results from the simulation study

The simulation result revealed that when N is small, the MAHAB estimator outperformed all other estimators for all the time periods except when N=10 and T=20, AH (d) estimator performed better than all other estimator for all the degrees of autocorrelation. When N=50, the MAHAB estimator performed better than all other estimators for T=5, but as T increases, the AH (d) estimator performed better than all other estimator for all the degrees of autocorrelation. When N=100, the MAHAB estimator outperformed all other estimators for time periods 5, 10 and 15 while ABGMM performed better than all other estimators as T increases to 20. But as n increases to 200, ABGMM outperformed all other estimators for all time periods and for all the degrees of autocorrelation in terms of absolute bias.

For the estimate of β, the simulation result showed that when n is small, the MAHAB estimator outperformed all other estimators for all the time periods except when n=20 and T=10 and 15, ABGMM performed better than all other estimators for all the degrees of autocorrelation. When n=50, the MAHAB estimator performed better than all other estimators when T=5 and 20 while the robust estimators (M and MM) performed better at T=10, and AH (d) estimator outperformed better than all other estimators when T=15. As n increases to 200, the MAHAB

estimator performed better than all other estimators for all the time periods and for all the degrees of autocorrelation in terms of absolute bias.

For the estimate of δ, the simulation result revealed that when n=10, AH (d) and ABGMM estimators performs better than all other estimator when T is 5. But as T increases, the MAHAB estimator performed better than all other estimators. When n=20, AH (d) estimator performed better than all other estimators when T=5 while the MAHAB estimator outperformed all other estimator when time periods is 10, 15 and 20. As n increases to 50, AH (d) estimator outperformed all other estimators at T=5.10 and 15 when ABGMM performed better than all other estimators when the time period is 20. When n is large, the MAHAB estimator outperformed all other estimators for all time periods except when n=100 and T=5 that AH (d) estimator performed better than all other estimators for all degrees of autocorrelation in terms of RMSE.

For the estimate of β, the simulation result revealed that when n is small, ABGMM outperformed all other estimators for time periods 5 and 10, while the proposed modified estimator performed better than all other estimators for time periods 15 and 20 for all the degrees of autocorrelation. As n increases, the MAHAB estimator outperformed all other estimators for all the time periods and for all the degrees of autocorrelation in terms of RMSE.

4 Conclusion

The Simulation results on various generating mechanism showed that based on a root mean squares error criterion, the MAHAB estimator performed well against the existing estimators.

Furthermore, the study also observed that when the value of the autoregressive parameter of the explanatory variable λ is varies, the absolute bias and RMSE of the estimators improves as the values of λ increases. The result of our findings showed that, as to be expected all estimators (with the exception Blundell-Bond System GMM and OLS) generally performed better with small T and large T. However, the MAHAB estimator seems to show the largest improvement as n and T increases.

This study concluded that in estimating the parameters of dynamic panel models in the presence of autocorrelation of the error term, the MAHAB estimator is more preferable. It is recommended to use the MAHAB estimator when dealing with panel data models in the presence of serial correlation.

Acknowledgements

Competing Interests

Authors have declared that no competing interests exist.

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Appendix

Table 1. RMSE of various estimators when n=100 and T=20 for model parameter (β)

λ	ρ	θ	ARMA(1,1) RMSE (β) T=20,			n=100	δ=0.5	MM-EST.	MAHAB
			OLS	ABGMM1	SYS 1	AH(d)	M-EST.		
0.3	0.2	0.2	0.036223	0.029003	0.202796	0.513665	0.038517	0.044968	0.016232
		0.5	0.036217	0.029003	0.179303	0.351116	0.038426	0.044971	0.016232
		0.9	0.036214	0.029003	0.416322	0.238191	0.038392	0.044891	0.016232
	0.5	0.2	0.036214	0.015211	0.251678	0.289181	0.040089	0.045773	0.016232
		0.5	0.036211	0.029003	0.286583	0.256675	0.039085	0.045754	0.016232
		0.9	0.036207	0.029003	0.306292	0.209408	0.038317	0.045667	0.016232
	0.9	0.2	0.036187	0.029003	0.063073	0.119409	0.043239	0.046883	0.016232
		0.5	0.036202	0.029003	0.331865	0.142532	0.043279	0.046173	0.016232
		0.9	0.036206	0.029003	0.401434	0.146873	0.040395	0.045912	0.016232
0.5	0,2	0.2	0.035407	0.029003	0.202796	0.470044	0.037941	0.044296	0.015757
		0.5	0.035401	0.029003	0.179303	0.339268	0.037927	0.044242	0.015757
		0.9	0.035396	0.029003	0.416322	0.237051	0.037873	0.044123	0.015758
	0.5	0.2	0.035401	0.015211	0.251678	0.278249	0.039034	0.045159	0.015753
		0.5	0.035396	0.029003	0.286583	0.250751	0.038354	0.045107	0.015755
		0.9	0.035392	0.029003	0.306290	0.208231	0.037845	0.044993	0.015753
	0.9	0.2	0.035381	0.029003	0.063073	0.124562	0.041466	0.045973	0.015750
		0.5	0.035393	0.029003	0.331865	0.145881	0.041533	0.045416	0.015758
		0.9	0.035392	0.029003	0.401434	0.147911	0.039417	0.045288	0.015756
0.7	0.2	0.2	0.034404	0.029003	0.202796	0.424753	0.037081	0.043218	0.015664
		0.5	0.034398	0.029003	0.179303	0.323621	0.037142	0.043114	0.015666
		0.9	0.034393	0.029003	0.416322	0.233611	0.037102	0.042962	0.015668
	0.5	0.2	0.034401	0.015214	0.251678	0.264403	0.037623	0.044056	0.015663
		0.5	0.034395	0.029003	0.286583	0.242064	0.037395	0.043995	0.015663
		0.9	0.034394	0.029003	0.306296	0.206128	0.037048	0.043868	0.015665
	0.9	0.2	0.034389	0.029003	0.063073	0.128213	0.039787	0.044593	0.015666
		0.5	0.034395	0.029003	0.331865	0.146347	0.039338	0.044242	0.015669
		0.9	0.034392	0.029003	0.401434	0.148135	0.038324	0.044208	0.015670

Table 2. RMSE of various estimators when n=100 and T=10 for model parameter (β)

λ	ρ	θ	AR(1) ABSOLUTE BIAS (δ) T=10			n=100	M-EST.	MM-EST.	P-EST
			OLS	ABGMM1	SYS 1	AH(d)			
0.3	0.3	0.2	0.010162	0.002366	0.000856	0.001943	0.006294	0.003315	0.000659
		0.5	0.014119	0.002366	0.000856	0.001439	0.005037	0.012855	0.000656
		0.9	0.017738	0.002366	0.000856	0.001375	0.011992	0.027358	0.000646
	0.5	0.2	0.011296	0.002366	0.000856	0.001778	0.009465	0.002811	0.000617
		0.5	0.015175	0.002366	0.000856	0.001407	0.001025	0.011307	0.000612
		0.9	0.018785	0.002366	0.000856	0.001355	0.013855	0.026603	0.000615
	0.7	0.2	0.012288	0.002366	0.000856	0.001704	0.013032	0.001193	0.000573
		0.5	0.016157	0.002366	0.000856	0.001403	0.002276	0.009166	0.000552
		0.9	0.019613	0.002366	0.000856	0.001339	0.014626	0.025221	0.000542
0.5	0.3	0.2	0.010752	0.001971	0.000917	0.002038	0.006631	0.003568	0.000493
		0.5	0.014645	0.001971	0.000917	0.001482	0.000812	0.010549	0.000489
		0.9	0.018336	0.001971	0.000917	0.001278	0.017411	0.027317	0.000452
	0.5	0.2	0.011918	0.001971	0.000917	0.001973	0.008596	0.003715	0.000359
		0.5	0.015792	0.001971	0.000917	0.001467	0.001243	0.010266	0.000345
		0.9	0.019141	0.001971	0.000917	0.001255	0.019169	0.026781	0.000303
	0.7	0.2	0.012915	0.001871	0.000917	0.001933	0.010972	0.003333	0.000303
		0.5	0.016797	0.001971	0.000917	0.001465	0.003106	0.009465	0.000301
		0.9	0.019852	0.001971	0.000917	0.001236	0.018329	0.025473	0.000305
0.7	0.3	0.2	0.011031	0.001583	0.000930	0.002018	0.005509	0.003682	0.000512
		0.5	0.014684	0.001583	0.000930	0.001522	0.002739	0.009607	0.000588
		0.9	0.018185	0.001583	0.000931	0.001179	0.011333	0.026221	0.000513
	0.5	0.2	0.012212	0.001583	0.000930	0.002001	0.007298	0.004016	0.000442
		0.5	0.015806	0.001583	0.000930	0.001523	0.004163	0.009485	0.000453

λ	ρ	θ	AR(1) ABSOLUTE BIAS (δ) T=10				n=100	M-EST.	MM-EST.	P-EST
			OLS	ABGMM1	SYS 1	AH(d)				
		0.9	0.018978	0.001583	0.000931	0.001156	0.014307	0.025271	0.000492
	0.7	0.2	0.013202	0.001583	0.000930	0.001994	0.009351	0.004129	0.000435
		0.5	0.016819	0.001583	0.000930	0.001528	0.005553	0.009283	0.000392
		0.9	0.019634	0.001583	0.000931	0.001138	0.011675	0.023964	0.000323

Table 3. RMSE of various estimators when n=200 and T=5 for β

λ	ρ	θ	ARMA(1,1) RMSE (β) T=5			δ=0.7	n=200	MM-EST.	P-EST.
			OLS	ABGMM1	SYS 1	AH(d)	M-EST.		
0.3	0.2	0.2	0.050181	0.060110	0.470631	0.070608	0.053775	0.057329	0.039618
		0.5	0.050169	0.052772	0.471389	0.043860	0.053737	0.057405	0.039618
		0.9	0.050156	0.044209	0.472447	0.035059	0.053627	0.057462	0.039618
	0.5	0.2	0.050208	0.058365	0.464691	0.097112	0.053760	0.057154	0.039618
		0.5	0.050198	0.051537	0.464238	0.060793	0.053906	0.057159	0.039618
		0.9	0.050171	0.043691	0.465746	0.029366	0.054139	0.057185	0.039618
	0.9	0.2	0.050239	0.051524	0.44427	0.079152	0.053737	0.057919	0.039618
		0.5	0.050218	0.046535	0.441243	0.064645	0.054305	0.057635	0.039618
		0.9	0.050203	0.040488	0.441876	0.036184	0.053827	0.057303	0.039618
0.5	0,2	0.2	0.049093	0.059604	0.440085	0.063842	0.052078	0.055762	0.038919
		0.5	0.049083	0.052221	0.443087	0.043794	0.052132	0.055906	0.038919
		0.9	0.049071	0.043808	0.445522	0.036846	0.051987	0.055992	0.038919
	0.5	0.2	0.049118	0.057677	0.442375	0.094627	0.052426	0.055493	0.038919
		0.5	0.049101	0.051017	0.443062	0.058544	0.052646	0.055646	0.038919
		0.9	0.049084	0.043243	0.444572	0.029869	0.052881	0.055748	0.038919
	0.9	0.2	0.049154	0.050873	0.421312	0.076898	0.052247	0.056183	0.038919
		0.5	0.049135	0.045976	0.420230	0.063577	0.052451	0.055973	0.038919
		0.9	0.049119	0.040014	0.422282	0.036950	0.052324	0.055799	0.038919
0.7	0.2	0.2	0.047660	0.059072	0.404408	0.057927	0.050339	0.053909	0.038828
		0.5	0.047651	0.051721	0.409247	0.044063	0.050149	0.054129	0.038828
		0.9	0.047641	0.043397	0.413338	0.039239	0.049947	0.054256	0.038828
	0.5	0.2	0.047682	0.056988	0.412498	0.087468	0.050878	0.053620	0.038828
		0.5	0.047666	0.050479	0.414724	0.054767	0.051064	0.053871	0.038828
		0.9	0.047651	0.042843	0.416590	0.029845	0.051075	0.054055	0.038828
	0.9	0.2	0.047722	0.050213	0.392906	0.072278	0.050511	0.054177	0.038828
		0.5	0.047703	0.045426	0.393619	0.060742	0.050535	0.054072	0.038828
		0.9	0.047687	0.039564	0.396951	0.036830	0.050567	0.054021	0.038828

Table 4. RMSE of various estimators when n=50 and T=5 for β

λ	ρ	θ	MA(1) RMSE (β) T=5, n=50			AH(d)	M-EST.	MM-EST.	P-EST.
			OLS	ABGMM1	SYS 1				
0.3	0.3	0.2	0.106431	0.116697	0.363142	0.952899	0.102783	0.102808	0.036214
		0.5	0.106475	0.106656	0.380534	0.464541	0.103419	0.102636	0.036003
		0.9	0.106515	0.092443	0.393002	0.253823	0.104602	0.102487	0.036413
	0.5	0.2	0.105835	0.116085	0.339367	0.926208	0.103167	0.101755	0.035281
		0.5	0.105895	0.105975	0.359424	0.499245	0.103604	0.101405	0.034137
		0.9	0.105944	0.091822	0.375828	0.281829	0.104418	0.101058	0.033125
	0.7	0.2	0.104415	0.115352	0.319733	0.848097	0.103128	0.099783	0.032173
		0.5	0.104481	0.105191	0.338098	0.511573	0.103029	0.099296	0.032065
		0.9	0.104529	0.091121	0.356224	0.302012	0.103461	0.103642	0.031068
0.5	0.3	0.2	0.106474	0.106831	0.374752	0.433557	0.103422	0.102564	0.037582
		0.5	0.106501	0.095521	0.392852	0.296849	0.104301	0.102480	0.035217
		0.9	0.106524	0.081447	0.407798	0.204885	0.105526	0.102513	0.034101
	0.5	0.2	0.105883	0.106041	0.353666	0.457262	0.103712	0.101332	0.033252
		0.5	0.105930	0.094727	0.373942	0.332076	0.104018	0.101074	0.033102
		0.9	0.105966	0.080776	0.392135	0.233323	0.104519	0.103415	0.032751
	0.7	0.2	0.104459	0.105140	0.332805	0.461632	0.102909	0.103484	0.031292
		0.5	0.104516	0.093853	0.352639	0.357631	0.103148	0.101179	0.031251

λ	ρ	θ	MA(1) RMSE (β) T=5, n=50			AH(d)	M-EST.	MM-EST.	P-EST.
			OLS	ABGMM1	SYS 1				
		0.9	0.104554	0.080049	0.372442	0.257731	0.102966	0.103273	0.030288
0.7	0.3	0.2	0.106511	0.096430	0.381164	0.253949	0.103816	0.102344	0.035178
		0.5	0.106519	0.085116	0.400609	0.213694	0.104713	0.102557	0.034008
		0.9	0.106532	0.071937	0.418106	0.168627	0.104943	0.102744	0.034005
	0.5	0.2	0.105921	0.095573	0.362341	0.280227	0.103739	0.107146	0.035888
		0.5	0.105952	0.084313	0.383555	0.245007	0.103917	0.105883	0.032881
		0.9	0.105981	0.071286	0.403856	0.195496	0.104767	0.106802	0.031872
	0.7	0.2	0.104490	0.094620	0.341966	0.298137	0.103025	0.098726	0.034186
		0.5	0.104535	0.083444	0.362811	0.271539	0.102972	0.104793	0.032525
		0.9	0.104569	0.070595	0.384546	0.220252	0.103080	0.099503	0.031293

Table 5. Absolute bias of various estimators when n=100 and T=20 for δ

λ	ρ	θ	ARMA(1,1) RMSE (β) T=20,			n=100	δ=0.7	MM-EST.	P-EST
			OLS	ABGMM1	SYS 1	AH(d)	M-EST.		
0.3	0.2	0.2	0.036210	0.028716	0.048747	0.316261	0.038486	0.044933	0.018188
		0.5	0.036212	0.028716	0.436517	0.253642	0.038371	0.044919	0.018188
		0.9	0.036215	0.028716	0.413555	0.197932	0.038048	0.044876	0.018188
	0.5	0.2	0.036204	0.028716	0.048747	0.222577	0.039391	0.045795	0.018188
		0.5	0.036206	0.028716	0.048747	0.206769	0.038532	0.045757	0.018188
		0.9	0.036209	0.028716	0.129234	0.182227	0.037806	0.045692	0.018188
	0.9	0.2	0.036186	0.028716	0.048747	0.109866	0.042538	0.046505	0.018188
		0.5	0.036204	0.028716	0.445736	0.128344	0.041234	0.045982	0.018188
		0.9	0.036209	0.028716	0.048747	0.138938	0.038373	0.045966	0.018188
0.5	0,2	0.2	0.035395	0.028716	0.048747	0.300971	0.037944	0.044237	0.018162
		0.5	0.035395	0.028716	0.436517	0.250854	0.037813	0.044176	0.018162
		0.9	0.035398	0.028716	0.413554	0.200442	0.037571	0.044085	0.018162
	0.5	0.2	0.035391	0.028716	0.048747	0.216153	0.038588	0.045174	0.018162
		0.5	0.035391	0.028716	0.048747	0.204766	0.038107	0.045103	0.018162
		0.9	0.035393	0.028716	0.129234	0.184393	0.037421	0.044994	0.018162
	0.9	0.2	0.035381	0.028716	0.048747	0.113274	0.040763	0.045736	0.018162
		0.5	0.035393	0.028716	0.445736	0.130694	0.040103	0.045333	0.018162
		0.9	0.035395	0.028716	0.048747	0.140536	0.038046	0.045371	0.018162
0.7	0.2	0.2	0.034392	0.028716	0.048747	0.284354	0.037089	0.043136	0.017292
		0.5	0.034392	0.028716	0.436517	0.245773	0.037053	0.043036	0.017292
		0.9	0.034394	0.028716	0.413554	0.201288	0.036809	0.042902	0.017292
	0.5	0.2	0.034391	0.028716	0.048747	0.208437	0.037486	0.044071	0.017292
		0.5	0.034393	0.028716	0.048747	0.202732	0.037166	0.043989	0.017292
		0.9	0.034391	0.028716	0.129234	0.185189	0.036766	0.043851	0.017292
	0.9	0.2	0.034386	0.028716	0.048747	0.116022	0.039132	0.044488	0.017292
		0.5	0.034394	0.028716	0.445736	0.132003	0.038733	0.044224	0.017292
		0.9	0.034396	0.028716	0.048747	0.140772	0.037117	0.044278	0.017292

Table 6. RMSE of various estimators when n=100 and T=20 for β

λ	ρ	θ	ARMA(1,1) ABSOLUTE BIAS (δ) T=20,			n=100	δ=0.3	MM-EST.	P-EST
			OLS	ABGMM1	SYS 1	AH(d)	M-EST.		
0.3	0.2	0.2	0.001414	0.000178	0.004742	0.000397	0.003522	0.005629	0.000274
		0.5	0.000197	0.000178	0.377190	0.000516	0.002615	0.006318	0.000274
		0.9	0.001405	0.000178	0.408636	0.000603	0.001208	0.006843	0.000274
	0.5	0.2	0.001248	0.000178	0.152399	0.000626	0.011992	1.26E-05	0.000274
		0.5	0.002250	0.000178	0.307599	0.000608	0.009138	0.003097	0.000274
		0.9	0.003446	0.000178	0.376933	0.000612	0.006097	0.005466	0.000274
	0.9	0.2	0.002352	0.008340	0.313771	0.000747	0.025008	0.013087	0.000274
		0.5	0.003586	0.000178	0.364291	0.000714	0.023539	0.006905	0.000274
		0.9	0.004746	0.000178	0.004742	0.000677	0.021665	0.001557	0.000274
0.5	0,2	0.2	0.001286	0.000178	0.004742	0.000459	0.001971	0.005913	0.000249
		0.5	2.82E-05	0.000178	0.377191	0.000537	0.002146	0.006215	0.000249

λ	ρ	θ	ARMA(1,1) ABSOLUTE BIAS (δ) T=20,			n=100	δ=0.3	MM-EST.	P-EST
			OLS	ABGMM1	SYS 1	AH(d)	M-EST.		
		0.9	0.001565	0.000178	0.408636	0.000611	0.001460	0.006789	0.000249
	0.5	0.2	0.001352	0.000178	0.152399	0.000635	0.009730	0.000879	0.000249
		0.5	0.002375	0.000178	0.307590	0.000614	0.008246	0.003523	0.000249
		0.9	0.003561	0.000178	0.376933	0.000618	0.005989	0.005738	0.000249
	0.9	0.2	0.002544	0.008342	0.313771	0.000743	0.024286	0.012577	0.000249
		0.5	0.003726	0.000178	0.364291	0.000709	0.021446	0.006024	0.000249
		0.9	0.004812	0.000178	0.004742	0.000676	0.019308	0.000735	0.000249
0.7	0.2	0.2	0.001125	0.000178	0.004742	0.000512	0.000749	0.006181	0.000215
		0.5	0.000156	0.000178	0.377192	0.000557	0.001257	0.005966	0.000215
		0.9	0.001733	0.000178	0.408636	0.000617	0.001046	0.006491	0.000215
	0.5	0.2	0.001498	0.000178	0.152399	0.000643	0.007883	0.001553	0.000215
		0.5	0.002512	0.000178	0.307590	0.000621	0.007294	0.003868	0.000215
		0.9	0.003688	0.000178	0.376933	0.000622	0.004832	0.005765	0.000215
	0.9	0.2	0.002745	0.008342	0.313771	0.000739	0.022681	0.011547	0.000215
		0.5	0.003787	0.000178	0.364291	0.000703	0.018943	0.005246	0.000215
		0.9	0.004905	0.000178	0.004742	0.000675	0.017043	0.000169	0.000215