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Calculation method for elastic parabolic cable subjected to uniformly distributed load on each segment and concentrated load at many points

Vu Khac Bảy[a], Duong Van Tai[b], Dong Van Ngoc[c], Luong Anh Tuan[d*], Hoang Nhan[a] and Hoang Ha[b]

[a]*Faculty of Engineering and Technology (FET), Nguyen Tat Thanh University, Ho Chi Minh City 700000, Vietnam*
[b]*College of Electromechanical and Civil Engineering, Vietnam National University of Forestry, Hanoi, Vietnam*
[c]*Hanoi College for Electro – Mechanics (HCEM), Vietnam*
[d]*Faculty of Fire Prevention, University of fire prevention and fighting, Hanoi, Vietnam*

A R T I C L E I N F O

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A B S T R A C T

Using the assumption that the load is evenly distributed in the horizontal direction, the article has given the cable deflection equation as a function that depends only on the horizontal coordinates, the length of the cable and horizontal distance between two supports. This result leads to the construction of a general system of equations to calculate the deflection, tension, and elongation of an elastic single cable resting on two supports with or without high difference, bearing uniformly distributed loads (or evenly distributed at intervals) and load is concentrated at many points. Calculations of examples to compare with results have been performed by other methods.

1. Introduction

Cables are used in many construction projects such as suspension bridges, domes, power transmission lines, cable car systems transporting people and goods, etc. Computational models of cables are also developed to suit the properties of each type of building in which it is applied and mainly develop in two directions:

1) The first direction: Using the finite molecule method, divide the cable into small segments, each cable segment is an element and the split points are nodes. The simplest element most commonly used in the analysis of cable structures is the two-node straight bar element having only axial stiffness (Ozdemir 1979). Elastic catenary cable formulation was first presented by O'Brien and Francis (1964) and Jayaraman and Knudson (1981), it requires less number of elements in cable structures modeling (Andreu et al. 2006, Yang and Tsay 2007). The parabolic elastic element is also used due to its simpler shape compared to catenary (Irvine 1992).
2) The second direction: Based on the assumption that the load is uniformly distributed in the horizontal direction, the deflection equation of the cable span has a parabolic shape.

* Corresponding author.
E-mail addresses: luonganhtuan@daihocpccc.edu.vn (T.U. Luan)

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A cable suspended horizontally under the influence of its own weight has the shape of a catenary. The mathematical solution of the catenary is attributed to James Bernoulli in 1691. In 1794, again in connection with the design of a proposed suspension bridge it was found that, if the cable's weight was assumed to be uniformly distributed along the span rather than along the cable, the cable hung in a parabolic profile. Ren et al. (2008), presented a finite element (FE) analysis of two-node parabolic cable element for analysis of cable structures with considering static behavior. They concluded that their results are in good agreement with the results obtained from theory of parabolic cable that considers the nonlinear effects. Rezaiee-Pajand et al. (2018) proposed a novel element for considering nonlinear thermo-elastic effects in cables. Abed et al. (2013) also analyzed the cable structures using nonlinear assumption for general loading cases. Some researchers implemented multiple-node (e.g. four and six node) isogeometric elements for analyzing and modeling of cables (Coyette & Guisset, 1988; Ali & Abdel-Ghaffar, 1995; Jian-hua & Wen-zhang, 2015; Wang et al. 2013). However, such models are relatively complex and with a large number of degrees of freedom. Several scholars and researchers have also modeled the cables subjected to different applied loadings (Jiang et al. 2022; Impollonia et al. 2011; Castro-Fresno et al.; 2008; Greco et al. 2014).

In some research works the parabola method has been utilized for the design, analysis and calculation of strength or deformation of cables (Ren et al. 2008; Tibret, 1999; Wang & Yang 1996). The parabolic cable has since received considerable attention, not only because of its simplicity, but also because in many situations (such as suspension bridges), a substantial part of the load is uniformly distributed along the span.

A inextensible cable hanging under its own weight, if the ratio of sag to span is 1:8, or less, the load may be assumed uniformly distributed along the span (Fig.1a), then the deflection equation will have the form of: $y=\frac{4f_c}{d}\left(x-\frac{x^2}{d}\right)$. For this cable, the horizontal component of cable tension is: $H=\frac{qd^2}{8f_c}$ and the longitudinal tension at any point in the cable is: $T=H\sqrt{1+\left[y'(x)\right]^2}$; where q is the weight of the cable per unit length, d is the cable span distance, f_c is the cable deflection at mid-span $\left(x=\frac{d}{2}\right)$. Also the length of the cable is determined from: $L=\int_0^d\sqrt{1+\left[\frac{4f_c}{d}\left(1-\frac{2x}{d}\right)\right]^2}\,dx$.
The concept of cable elasticity received little attention until 1858 when Rankine gave an approximate solution for the increase in sag obtained when an inextensible, free-hanging parabolic cable is allowed to stretch. Then, it was not until 1891 that Routh gave the solution of the elastic catenary. However, Rankine's solution contains unnecessary approximations, while Routh's solution is inconvenient on account of the coordinate system used.

In 1974, Irvine (1974) came up with a solution to solve the limitations of the above two approaches. By calculating the value of the ratio $\frac{f_c}{d}$ in terms of the approximate value of the length L over the polynomial expansion, Irvine gave the additional deflection (due to the elasticity of the cable) found to be $v=\frac{\Delta H}{H-\Delta H}\frac{qd^2}{2H}\left\{\frac{x}{d}-\left(\frac{x}{d}\right)^2\right\}$, where ΔH is the amount of reduction of the horizontal component of the cable tension.

In this paper, an approach is proposed for the calculation method of elastic parabolic cables, which are subjected to many concentrated loads and at the same time are subjected to uniform loads over each interval. Unlike Irvine, from the exact calculation of the parameter $u=\frac{4f_c}{d}$ value according to the cable length, L; leads to the cable deflection equation is determined when knowing the distance between the two supports, d , and the parameter u .

In the case of a cable span with a high difference between the two supports, h_s , with the idea of considering the span of the cable as part of a hypothetical cable span with two supports with no difference in height, it is easy to get the deflection equation of the cable span is expression depends on the parameters u, d, h_s.

This result is used to build a calculation method for elastic cable spans that are subjected to many concentrated loads and at the same time are subjected to uniform loads on each cable segment. In this case, each cable segment can be considered as a cable span characterized by: h_i , d_i are the height difference, the horizontal distance of the two supports and the parameter u_i , respectively. From the condition of force balance at the junctions between spans, the system of nonlinear equations of variables: u_i, h_i, d_i is obtained. By solving this system of equations according to the Newton-Raphson method the values of quantities of the tensile force along the string, T ; horizontal tension component, H ; displacements at the loading points are then calculated.

Thus, by the calculation method for the cases of elastic cable spans under uniformly distributed loads, the article builds a general system of equations to calculate the deflection for elastic cable spans subjected to many concentrated loads and at the same time are subjected to uniform loads on each segment, resting on two supports with or without a height difference.

2. Equation of deflection of a single cable

2.1. Single cable with uniform load

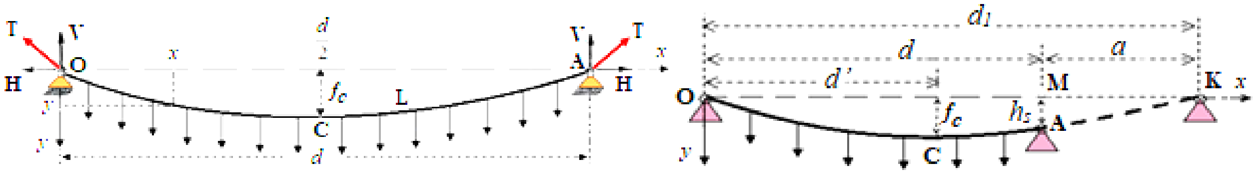

Fig. 1. Model of single cable with uniformly distributed load, resting on two supports:
(a) no high difference, (b) high difference h_s .

Single cable of length $L(m)$ rests on supports O and A with horizontal distance $d(m)$, no high difference (Fig. 1.a) or high difference (Fig.1.b). Select the coordinate system Oxy with origin O coincides with a support (in case two supports are of equal height) or origin O coincides with a support with higher elevation (in case of high difference), the Ox axis is horizontal and lying. In the vertical plane connecting the two supports the Oy axis points downward. Assume the load is uniformly distributed in the horizontal direction with the strength $q\left(N/m\right)$. Table 1 also describes the symbols used in the current research.

Table 1. Symbols of quantities

Content	Symbol	Unit
The cable span length	L	m
The cable span distance (Horizontal distance between two supports)	d	m
The sag (Maximum deflection of the cable)	f_c	m
The load intensity evenly distributed along the length of the cable	γ	N/m
The load intensity is evenly distributed in the horizontal direction Ox	q	N/m
The tension in the cable	T	N
The horizontal component of cable tension	H	N
The vertical component of the cable tension	V	N
Height difference between two supports	h_s	m
Horizontal distance between low bearing (A) and assumed support (K) (Fig. 2)	a	m
Horizontal distance between high bearing (O) and assumed support (K) (Fig. 2)	d_1	m
Young's modulus of elasticity	E	N/m^2
Cable cross-sectional area	F	m^2
The elongation of cable span	ΔL	m

2.1.1 Single cable with uniformly distributed load, resting on two pillows with no height difference

2.1.1.1 Equation of deflection

The cable of length $L(m)$, resting on two O, A supports, has no height difference (Fig. 1.a). Assume the cable is subjected to a uniformly distributed load and is inextensible cable.
From the condition that the moment at any point $\left(x, y\right)$ on the cable is zero, the force balance in the y direction (Fig. 1.a) is obtained:

$$-Hy + Vx - \int_0^x q(x-t)dt = 0 \tag{1}$$

$$V = \frac{qd}{2} \tag{2}$$

Leads to the deflection equation
$$y = \frac{qx(d-x)}{2H} \tag{3}$$

The cable deflection at $x = \frac{d}{2}$ is the sag and the horizontal component of cable tension is

$$f_c = \frac{qd^2}{8H} \;\; ; \;\; H = \frac{qd^2}{8f_c} \tag{4}$$

Substituting (4) in (3) get the deflection equation:

$$y = \frac{4f_c}{d}\left(x - \frac{x^2}{d}\right) \tag{5}$$

The deflection equation of a single cable in the form Eq. (5) was also presented by Irvine (1974). However, Eq. (5) also contains the undefined quantity f_c. Symbol $u = \frac{4f_c}{d}$, the deflection equation Eq. (5) becomes:

$$y = u\left(x - \frac{x^2}{d}\right). \tag{6}$$

If point $M(x,y)$ lies on the cable, the length of the OM cable will be:

$$L_x = \int_0^x \sqrt{1+[y'(t)]^2}\, dt = \int_0^x \sqrt{1+\frac{u^2}{d^2}(2t-d)^2}\, dt \tag{7}$$

Integral Eq. (7) will get:

$$L_x = \frac{d}{4}\left\{\sqrt{1+u^2} + \frac{\ln\left(u+\sqrt{1+u^2}\right)}{u} + B\sqrt{1+(Bu)^2} + \frac{\ln\left(Bu+\sqrt{1+(Bu)^2}\right)}{u}\right\} \tag{8}$$

where $B = \frac{2x-d}{d}$. When $x = d$, then $B = 1$, resulting in the length of the span's cable segment:

$$L = \frac{d}{2}\left\{\sqrt{1+u^2} + \frac{\ln\left(u+\sqrt{1+u^2}\right)}{u}\right\} \tag{9}$$

Symbol $A = \frac{2L}{d} > 2$, the Eq. (9) is equivalent to:

$$\mathrm{W}(u) = \sqrt{1+u^2} + \frac{\ln\left(u+\sqrt{1+u^2}\right)}{u} - A = 0 \tag{10}$$

Symbol $a = \sqrt{A^2-2A} > 0$, $b = \sqrt{A^2-1} > 0$ found that $\mathrm{W}'(u) = \frac{1}{u}\left[\sqrt{1+u^2} - \frac{\ln\left(u+\sqrt{1+u^2}\right)}{u}\right] > 0 \;\forall\; u > 0$,

$W(a) < 0$ and $W(b) > 0$, so equation $W(u) = 0$ will exist and have a unique solution on $(a,\, b)$. The finding of the solution u will be by the method of consecutively bisecting the segment $[a,b]$.

Eq. (6) is the equation representing the deflection of the span of the cable resting on two supports without height difference, assuming the load is uniformly distributed in the horizontal direction and the cable has no stretch. Where u is the solution of equation (10).

2.1.1.2 Horizontal tension and longitudinal tensile force in the cable

The horizontal component of cable tension is

$$H = \frac{qd^2}{8f_c} = \frac{qd}{2u} \tag{11}$$

The tension at any point $M(x, y)$ in the cable is

$$T = H\sqrt{1+\left[y'(x)\right]^2} = \frac{qd}{2u}\sqrt{1+\frac{u^2}{d^2}(d-2x)^2} \tag{12}$$

2.1.1.3 The elongation of cable span

The tensile stress $\sigma(N/m^2)$ of the cable is a function of x as below equation:

$$\sigma(x)=\frac{T(x)}{F}=\frac{qd}{2F.u}\sqrt{1+\frac{u^2}{d^2}(d-2x)^2} \tag{13}$$

As a result, the element of length Δs at point x will have elongation length of $\Delta\varepsilon=\frac{\sigma(x)}{E}\Delta s$. Therefore, the cable length has increased by an amount

$$\Delta L=\int_0^L \frac{\sigma(x)}{E}ds = \frac{qd}{2E.F.u}\int_0^d \sqrt{1+\frac{u^2}{d^2}(d-2x)^2}\,.\sqrt{1+y'^2}dx = \frac{qd^2}{2E.F.u}\left[1+\frac{u^2}{3}\right] \tag{14}$$

To get the cable deflection equation including the wire elongation, it is necessary to solve Eq. (10) with the value of A replaced by $\overline{A}=\frac{2(L+\Delta L)}{d}$ to get the solution $\overline{u}$. Then the equation of deflection of the extensible cable, resting on two supports with no difference in height, with a uniformly distributed load in the horizontal direction is obtained from:

$$y=\overline{u}\left(x-\frac{x^2}{d}\right) \tag{15}$$

2.1.2 Single cable with uniformly distributed load, resting on two supports with high difference

2.1.2.1 Equation of deflection

The cable of length $L(m)$, resting on two O, A supports, has height difference $h_s(m)$ (Fig. 1.b). Assume the cable is subjected to a uniformly distributed load and is inextensible cable. Found that, the OA cable segment is part of the OAK cable span, with support K having the same height as support A. The cable span OAK is called assumed cable span. Choose the coordinates whose origin coincides with the support with the higher height. There is the coordinates of the points $A(d, h_s)$ and $K(d_1, 0)$.

Let f_c be the deflection (at the mid-span point) of the assumed cable span, then according to (5), the deflection equation of the cable span is obtained as

$$y=\frac{4f_c}{d_1}\left(x-\frac{x^2}{d_1}\right) \tag{16}$$

Since the length of cable segment OA is equal to L, there will be:

$$L=\int_0^d \sqrt{1+y'^2}\;dx=\int_0^d \sqrt{1+\frac{16f_c^2}{d_1^4}(d_1-2x)^2}\;dx \tag{17}$$

By defining symbol $a = d_1 - d$ and integrating Eq. (17) we get:

$$L=\frac{(d+a)^2}{8f_c}\left\{\frac{2f_c}{(d+a)}\sqrt{1+\frac{16f_c^2}{(d+a)^2}}+\frac{1}{2}ln\left|\frac{4f_c}{(d+a)}+\sqrt{1+\frac{16f_c^2}{(d+a)^2}}\right|\right.$$
$$\left.-\frac{2f_c(a-d)}{(d+a)^2}\sqrt{1+\frac{16f_c^2(a-d)^2}{(d+a)^4}}-\frac{1}{2}ln\left|\frac{4f_c(a-d)}{(d+a)^2}+\sqrt{1+\frac{16f_c^2(a-d)^2}{(d+a)^4}}\right|\right\} \tag{18}$$

where $u=\dfrac{4f_c}{d+a}$, the deflection equation (16) becomes: $y = u.\dfrac{x(d+a-x)}{(d+a)}$ (19)

There are:

$$y(d)=h_s=u.\frac{d(d+a-d)}{(d+a)}=u.\frac{d\,a}{(d+a)} \tag{20}$$

$$a=\frac{h_s d}{ud-h_s}\ ;\quad a-d=\frac{2h_s d-ud^2}{ud-h_s}\ ;\quad a+d=\frac{ud^2}{ud-h_s}\ ; \tag{21}$$

By assuming symbol $C(u) = \dfrac{a-d}{a+d} = \dfrac{2h_s d - ud^2}{ud^2} = \dfrac{2h_s}{ud} - 1$, from Eq. (18) leads to

$$L=\frac{d}{2(1-C)}\left\{\sqrt{1+u^2}+\frac{\ln\left(u+\sqrt{1+u^2}\right)}{u}-C\sqrt{1+C^2u^2}-\frac{\ln\left(Cu+\sqrt{1+C^2u^2}\right)}{u}\right\} \tag{22}$$

Find the solution u from the equation:

$$\Psi(u)=\frac{d}{2(1-C)}\left\{\sqrt{1+u^2}+\frac{\ln\left(u+\sqrt{1+u^2}\right)}{u}-C\sqrt{1+C^2u^2}-\frac{\ln\left(Cu+\sqrt{1+C^2u^2}\right)}{u}\right\}-L=0 \tag{23}$$

Substituting the value u found in Eq. (19) and Eq. (21), we get the equation of deflection of the inextensible cable OA, resting on two supports with difference in height h_s, with a uniformly distributed load in the horizontal direction as:

$$y=u\left(x-\frac{x^2}{d}\right)+\frac{h_s x^2}{d^2} \tag{24}$$

2.1.2.2 Finding the solution u of the equation $\Psi(u)=0$

Symbol $u_1=\dfrac{h_s}{d},\ u_0=\dfrac{2h_s}{d},\ u_2=\dfrac{\sqrt{16L^2-d^2}}{d}$. There is $1-C(u_1)=2-\dfrac{2h_s}{u_1 d}=0$,

so $u=u_1$ is the discontinuity of the function $\Psi(u)$

$$\text{There are}\quad h_s<2h_s=\sqrt{4h_s^2}<\sqrt{3h_s^2+L^2-d^2}<\sqrt{3L^2+L^2-d^2}<\sqrt{4L^2-d^2}<\sqrt{4L^2-\frac{d^2}{4}} \tag{25}$$

$$\Rightarrow\quad u_1=\frac{h_s}{d}<u_0=\frac{2h_s}{d}<\frac{2}{d}\sqrt{4L^2-\frac{d^2}{4}}=\frac{\sqrt{16L^2-d^2}}{d}=u_2\quad\Rightarrow\quad u_1<u_0<u_2$$

$$\text{There are}\quad \lim_{u\to u_1}\Psi(u)=\frac{d}{2}\left[\frac{1+2u_1}{\sqrt{1+u_1^2}}+\frac{1}{\sqrt{1+u_1^2}}\right]-L=d\sqrt{1+u_1^2}-L=\sqrt{d^2+h_s^2}-L<0 \tag{26}$$

Symbol: $$\overline{\Psi}(u) = \begin{cases} \Psi(u) & , \ u > u_1 \\ \sqrt{d^2 + h_s^2} - L & , u = u_1 \end{cases} \tag{27}$$

=> function $\overline{\Psi}(u)$ is continuous $\forall u \geq u_1$

Considering $u > u_0$: Because $C(u) = \frac{2h_s}{ud} - 1$

should $-1 < C < 0$ and $1 < 1 - C < 2 \Rightarrow \frac{d}{2(1-C)}\left[\frac{1}{u}\ln\frac{u+\sqrt{1+u^2}}{Cu+\sqrt{1+C^2u^2}} - C\sqrt{1+C^2u^2}\right] > 0$

$$\Rightarrow \overline{\Psi}(u) = \frac{d}{2(1-C)}\sqrt{1+u^2} + \frac{d}{2(1-C)}\left[\frac{1}{u}\ln\frac{u+\sqrt{1+u^2}}{Cu+\sqrt{1+C^2u^2}} - C\sqrt{1+C^2u^2}\right] - L > \frac{d}{4}\sqrt{1+u^2} - L \tag{28}$$

Therefore $$\overline{\Psi}(u_2) > \frac{d}{4}\sqrt{1+u_2^2} - L = 0 \tag{29}$$

The function $\overline{\Psi}(u)$ is continuous on $[u_1, \ u_2]$, satisfying the conditions: $\overline{\Psi}(u_1) = \sqrt{d^2+h_s^2} - L < 0$ and $\overline{\Psi}(u_2) > 0$, so there will exist and only solutions $\overline{\Psi}(u) = 0$ on $[u_1, \ u_2]$. The finding the solution u from equation (23) is done by the method of consecutive halving the segment $[u_1 \ , \ u_2]$.

2.1.2.3 Horizontal component of cable tension and tensile force along the wire

The horizontal component of cable tension $H = \frac{qd_1}{2u} = \frac{q(d+a)}{2u}$,

After substitution $a = \frac{h_s d}{ud - h_s}$ the following equation is obtained:

$$H = \frac{qd^2}{2(ud - h_s)} \tag{30}$$

The tensile force along the cable $T = H\sqrt{1+y'^2}$. From Eq. (24) one gets:

$y' = u - \frac{2ux}{d} + \frac{2h_s x}{d^2} \Rightarrow y'(0) = u$, $y'(d) = \frac{2h_s}{d} - u$, from which:

$$\begin{aligned} T &= H\sqrt{1+u^2} && \textit{at high cable support} \\ T &= H\sqrt{1+\left(u - \frac{2h_s}{d}\right)^2} && \textit{at low cable support} \end{aligned} \tag{31}$$

2.1.2.4 The elongation of cable span

The tensile stress along the cable is a function of x as below equation:

$$\sigma(x) = \frac{T(x)}{F} = \frac{qd^2}{2F.(ud - h_s)}\sqrt{1+\left[u - \frac{2ux}{d} + \frac{2h_s x}{d^2}\right]^2} \tag{32}$$

from which, the elongation of the cable in the span will be

$$\Delta L = \int_0^L \frac{\sigma(x)}{E}\, ds = \frac{qd^2}{2E.F.(ud - h_s)}\left[d + \frac{du^2}{3} + \frac{h_s}{3d}(4h_s - 2ud)\right] \tag{33}$$

If the elongation of the cable is taken into account, it is necessary to solve Eq.(23) with the value of L being replaced by $\overline{L} = L + \Delta L$ to obtain the solution $\overline{u}$ .Then the equation of deflection of the extensible cable, resting on two supports with difference in height h_s, load evenly distributed in the horizontal direction:

$$y=\bar{u}\left(x-\frac{x^2}{d}\right)+\frac{h_s x^2}{d^2} \tag{34}$$

If the height difference is equal to zero ($h_s = 0$), the calculation results for the cable span resting on supports with high difference return to the calculated results for the cable span when the supports are not difference in height.

2.2. Single cable under uniformly distributed and concentrated loads

2.2.1 Single cable under uniformly distributed load and only one concentrated load, resting on two supports with high difference

The assumption in this section is that the load is uniformly distributed horizontally with intensity $q(N/m)$. The load $Q(N)$ is concentrated at the point $M(x, y)$ whose height is lower than the two supports (as shown in Fig. 2a) or higher than the low support (Fig. 2b). The OA cable segment can be considered to be supported by three supports: O, M, A. If the values of parameters u_1, u_2 of the cable segments (OM, MA) and the deflection h_1 at M can be calculated, then according to section 2.1.2, the deflection equation of the cable segments OM and MA will be determined.

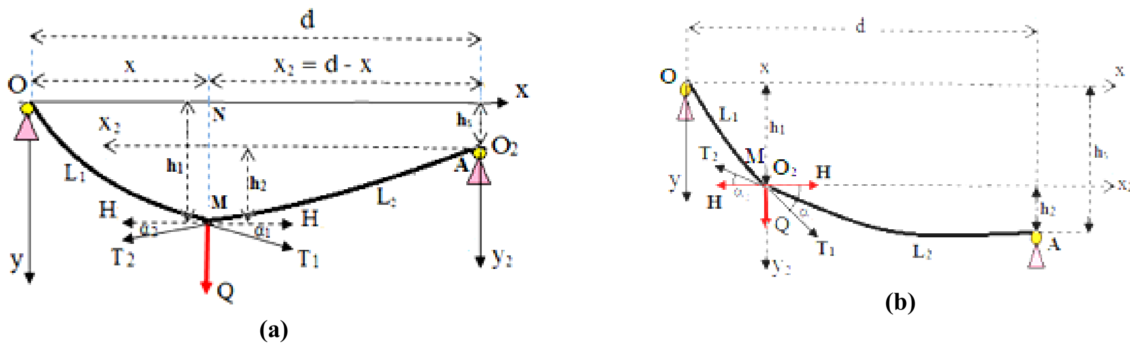

Fig. 2. Model of a single cable span under uniformly distributed load and only one concentrated load, resting on two supports with high difference.
(a) – Concentrated load application point is lower than the two supports, (b) – Concentrated load application point is higher than the low support

Let T_1, T_2 be the longitudinal tension forces of the cable segments L_1, L_2 at the point of applying concentrated load M. On the MA cable segment, take the $O_2x_2y_2$ coordinate system so that the origin of O_2 coincides with the point of higher elevation (with A in Fig. 2a or with M in Fig. 2b). The forces T_1, T_2 make up with the horizontal the corresponding angles α_1, α_2.

Let $d_1 = x$, $d_2 = d - x$ and u_1, u_2 be the variables in the deflection equation (according to Eq.(24)), respectively the cable segments L_1, L_2. Use Eq.(30) applied to cable segments L_1, L_2 to obtain:

$$H=\frac{qd_1^2}{2(u_1d_1-h_1)}=\frac{qd_2^2}{2(u_2d_2-h_2)} \tag{35}$$

Due to $h_2 = |h_1 - h_s| = (h_1 - h_s).sign(h_1 - h_s)$, Eq.(35) is equivalent to :

$$u_1d_1d_2^2+h_1\left[sign(h_1-h_s)d_1^2-d_2^2\right]-u_2d_2d_1^2-h_sd_1^2sign(h_1-h_s)=0 \tag{36}$$

By balancing the forces at M one gets:

$$H=T_1\cos\alpha_1=T_2\cos\alpha_2 \tag{37}$$

$$T_1\sin\alpha_1+T_2\sin\alpha_2=Q \tag{38}$$

$$H(\tan\alpha_1+\tan\alpha_2)=Q \tag{39}$$

Also from Eq.(24)it can be found:

$$\tan\alpha_1 = y_1'(d_1) = \frac{2h_1}{d_1} - u_1 \tag{40}$$

$$\tan(\alpha_2) = \begin{cases} \dfrac{2h_2}{d_2} - u_2 \ , \ h_1 > h_s \\ -u_2 \ , \ h_1 \le h_s \end{cases} \tag{41}$$

or $$\tan(\alpha_2) = (h_1 - h_s).\frac{\left[sign(h_1 - h_s) + 1\right]}{d_2} - u_2 \tag{42}$$

With the aid of Eq. (35), Eq. (40), Eq. (42) , Eq. (39) is more conveniently written as:

$$h_1\left\{2d_1d_2 + d_1^2\left[sign(h_1 - h_s) + 1\right] + \frac{2Q}{q}d_2\right\} - u_1\left\{d_1^2 d_2 + \frac{2Q}{q}d_2 d_1\right\} - u_2 d_1^2 d_2 - d_1^2\left[sign(h_1 - h_s) + 1\right]h_s = 0 \tag{43}$$

Applying Eq. (22) to the OM and MA cable segments result the below relations:

$$\begin{aligned} &-4L_1\left(u_1d_1 - h_1\right) + u_1d_1^2\sqrt{1+u_1^2} + d_1^2\ln\left(u_1 + \sqrt{1+u_1^2}\right) + \\ &\quad + \left(u_1d_1 - 2h_1\right)\sqrt{d_1^2 + \left(u_1d_1 - 2h_1\right)^2} + d_1^2\ln\frac{\left(u_1d_1 - 2h_1\right) + \sqrt{d_1^2 + \left(u_1d_1 - 2h_1\right)^2}}{d_1} = 0 \end{aligned} \tag{44}$$

$$\begin{aligned} &-4L_2\left[u_2d_2 - h_2\right] + u_2d_2^2\sqrt{1+u_2^2} + d_2^2\ln\left(u_2 + \sqrt{1+u_2^2}\right) + \\ &\quad + \left[u_2d_2 - 2h_2\right]\sqrt{d_2^2 + \left[u_2d_2 - 2h_2\right]^2} + d_2^2\ln\frac{\left[u_2d_2 - 2h_2\right] + \sqrt{d_2^2 + \left[u_2d_2 - 2h_2\right]^2}}{d_2} = 0 \end{aligned} \tag{45}$$

$$h_2 = \left|h_1 - h_s\right| = \left(h_1 - h_s\right)sign\left(h_1 - h_s\right) \tag{46}$$

$$d_1 + d_2 - d = 0 \tag{47}$$

Solve the system of 6 equations: (i.e. Eq. (36), Eq. (43), Eq. (44), Eq. (45), Eq. (46), Eq. (47)) to find the $d_1, d_2, u_1, u_2, h_1, h_2$ values according to the parameters L_1, L_2, d, Q, q, h_s. Solving the above system of equations is done by Newton-Raphson method. The initial values $d_1^{(0)}, d_2^{(0)}, u_1^{(0)}, u_2^{(0)}, h_1^{(0)}, h_2^{(0)}$ are calculated according to the section 2.1.2 applied to the cable segments OA, OM, MA when there is no load Q.

If the longitudinal extension of the cable is taken into account, then after calculating the values of u_1, u_2, h_1, h_2 from the above equations, Eq.(33) can be applied to calculate the elongation on the cable segments OM and MA respectively:

$$\Delta L_1 = \frac{qd_1^2}{2E.F.\left(u_1 d_1 - h_1\right)}\left[d_1 + \frac{d_1u_1^2}{3} + \frac{h_1}{3d_1}\left(4h_1 - 2u_1d_1\right)\right] \tag{48}$$

$$\Delta L_2 = \frac{qd_2^2}{2E.F.\left(u_2 d_2 - h_2\right)}\left[d_2 + \frac{d_2u_2^2}{3} + \frac{h_2}{3d_2}\left(4h_2 - 2u_2d_2\right)\right] \tag{49}$$

Resolving the system of six equations (i.e. (36), (43), (44), (45), (46), (47)) with the value L_i replaced by $\overline{L_i} = L_i + \Delta L_i,\ i = \overline{1,2}$, get the solution $\overline{d}_1, \overline{d}_2, \overline{u}_1, \overline{u}_2, \overline{h}_1, \overline{h}_2$. From here, the deflection equations for each cable segment A and B are obtained as described by Eq. (34):

$$y_i = \bar{u}_i\left(x_i - \frac{x_i^2}{d_i}\right) + \frac{h_i x_i^2}{d_i^2} \ , \ i = 1,2 \tag{50}$$

Tensile force along the cable is determined from:

$$\begin{aligned} T_o &= H\sqrt{1+u_1^2} && at\ O \\ T_A &= H\sqrt{1+\left\{(h_s-h_1)\frac{\left[sign(h_s-h_1)+1\right]}{d_2}-u_2\right\}^2} && at\ A \end{aligned} \tag{51}$$

2.2.2 The elastic single cable subjected to a load evenly distributed over each segment and a concentrated load at many points

Consider an elastic single cable resting on two supports O, A with the difference in height equal to $h_s\,(m)$. At points $M_i(x_i, y_i)$ on the cable is subjected to concentrated load $Q_i(N)$ $\left(i = \overline{1 \div n}\right)$ as shown in Fig. 3. On segments $M_{i-1}M_i$, load evenly is distributed along the cable with intensity $\gamma_i\,(N/m)$. Knowing the coordinates of the $M_i(x_i, y_i)$ points before placing the load, it is necessary to determine the coordinates of the $M_i(x_i, y_i)$ points and also the deflection equation of each segment of $M_{i-1}M_i$ cable after placing the load. Table 2 illustrates the symbols and parameters for the investigated cable.

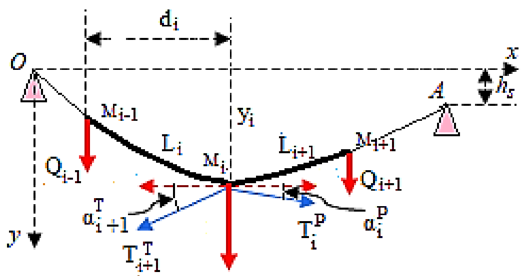

Fig. 3. Model of single cable subjected to a load evenly distributed over each segment and a concentrated load at many points.

Table 2. Symbols of quantities for defining the conditions of a single cable subjected to an evenly distributed load over each segment.

Content	Symbol	Unit
The length of the $M_{i-1}M_i$ cable segment, $i = \overline{1 \div n+1}$	L_i	m
The load intensity is uniform on the wire of the $M_{i-1}M_i$ cable segment.	γ_i	N/m
The projection of M_i on Ox	N_i	
The length of $N_{i-1}N_i$, $i = \overline{1 \div n+1}$	d_i	m
The deflection at M_i, $i = \overline{1 \div n+1}$	y_i	m
Tension force in cable segment $M_{i-1}M_i$ at M_i, $i = \overline{1 \div n}$	T_i^P	N
Tension force in cable segment M_iM_{i+1} at M_i, $i = \overline{1 \div n}$	T_{i+1}^T	N
The difference in height between two points M_{i-1} and M_i, $i = \overline{1 \div n+1}$.	$h_i = \lvert y_i - y_{i-1} \rvert$	m
Angle formed by T_i^P with the horizontal. $i = \overline{1 \div n}$	α_i^P	rad
Angle formed by T_{i+1}^T with the horizontal, $i = \overline{1 \div n}$	α_{i+1}^T	rad

where $M_0 \equiv O$; $M_{n+1} \equiv A$, $y_0 = 0$, $y_{n+1} = h_s$. The assumption is that on each segment $M_{i-1}M_i$ the load is uniformly distributed horizontally with intensity of $q_i = \gamma_i \dfrac{L_i}{d_i}(N/m)$.

Use Eq. (30) applied to cable segments (L_i and L_{i+1}) at points M_i $(i=\overline{1,n})$ to obtain:

$$H = \frac{q_i d_i^2}{2(u_i d_i - h_i)} = \frac{q_{i+1} d_{i+1}^2}{2(u_{i+1} d_{i+1} - h_{i+1})} \tag{52}$$

Substituting $h_i = |y_i - y_{i-1}| = (y_i - y_{i-1})\, sign(y_i - y_{i-1})$ into Eq. (52) will get:

$$\Phi_i = [u_{i+1} d_{i+1} - (y_i - y_{i+1}) sign(y_i - y_{i+1})] q_i d_i^2 - [u_i d_i - (y_i - y_{i-1}) sign(y_i - y_{i-1})] q_{i+1} d_{i+1}^2 = 0\ ,\ i = \overline{1,n} \tag{53}$$

Vertical and horizontal equilibriums at M_i lead to below equations:

$$\mathrm{T}_i^P \cos\alpha_i^P = T_{i+1}^T \cos\alpha_{i+1}^T = H \tag{54}$$

$$T_i^P \sin\alpha_i^P + T_{i+1}^T \sin\alpha_{i+1}^T = Q_i \tag{55}$$

$$H\left(\tan\alpha_i^P + \tan\alpha_{i+1}^T\right) = Q_i \tag{56}$$

On each cable segment $M_{i-1}M_i$ with length L_i considered in the local coordinate system $O_i\xi_i\eta_i$ ($O_i \equiv M_k$, $k \in \{i-1,\ i\}$, $y_k = \min\{y_{i-1}, y_i\}$), the deflection equation in the form of Eq. (24) is obtained as:

$$\eta_i = u_i\left(\xi_i - \frac{\xi_i^2}{d_i}\right) + \frac{h_i \xi_i^2}{d_i^2} \tag{57}$$

Hence get:

$$\tan(\alpha_i^P) = \frac{(y_i - y_{i-1})\left[sign(y_i - y_{i-1}) + 1\right]}{d_i} - u_i \tag{58}$$

$$\tan(\alpha_{i+1}^T) = \frac{(y_i - y_{i+1})\left[sign(y_i - y_{i+1}) + 1\right]}{d_{i+1}} - u_{i+1} \tag{59}$$

With the aid of Eq. (52), the Eq.56 is written as:

$$\Psi_i = d_i^2\left(\tan\alpha_i^P + \tan\alpha_{i+1}^T\right) - 2\frac{Q_i}{q_i}[u_i d_i - (y_i - y_{i-1}).sign(y_i - y_{i-1})] = 0\ ,\ i = \overline{1,n} \tag{60}$$

Applying the Eq. (22) to $M_{i-1}M_i$ cable segments will result:

$$W_i = \frac{d_i}{2(1-C_i)}\left\{\sqrt{1+u_i^2} + \frac{\ln\left(u_i + \sqrt{1+u_i^2}\right)}{u_i} - C_i\sqrt{1+C_i^2 u_i^2} - \frac{\ln\left(C_i u_i + \sqrt{1+C_i^2 u_i^2}\right)}{u_i}\right\} - L_i = 0,\quad i = \overline{1,n+1} \tag{61}$$

and
$$\Gamma = \sum_{i=1}^{n+1} d_i - d = 0 \tag{62}$$

where : $C_i = \dfrac{2(y_i - y_{i-1})\, sign(y_i - y_{i-1})}{u_i d_i} - 1$

Solve the system of $3n+2$ equations (53), (60), (61), (62) for $3n+2$ values of $y_1,...,y_n$, $d_1,...d_{n+1}$ and $u_1,...,u_{n+1}$. The solution of the above system of equations is done by Newton-Raphson method. The initial values $y_1^{(0)},...,y_n^{(0)}, d_1^{(0)},...,d_{n+1}^{(0)}, u_1^{(0)},...,u_{n+1}^{(0)}$ calculated according to section 2.1.2 are applied to the OA and $M_{i-1}M_i$ cable segments in the absence of the $Q_i, i=\overline{1,n}$ and $q_i, i=\overline{1,n+1}$ loads.

If the longitudinal extension of the cable is taken into account, then after calculating the values of $y_1,...,y_n$, $d_1,...d_{n+1}$ and $u_1,...,u_{n+1}$ from the above equations, Eq. (33) is applied to calculate the elongation on the cable segments $M_{i-1}M_i\left(i=\overline{1,n+1}\right)$ respectively:

$$\Delta L_i = \frac{q_i d_i^2}{2E.F.\left(u_i d_i - \left|y_i - y_{i-1}\right|\right)}\left[d_i + \frac{d_i u_i^2}{3} + \frac{\left|y_i - y_{i-1}\right|}{3d_i}\left(4\left|y_i - y_{i-1}\right| - 2u_i d_i\right)\right] \tag{64}$$

Resolving the system of $3n+2$ equations (53), (60), (61) and (62) with the value L_i replaced by $\overline{L_i} = L_i + \Delta L_i$, $i=\overline{1,\text{n}+1}$, get the solution $\overline{y}_i$, $i=\overline{0,n+1}$ and $\overline{u}_i$,$\overline{d}_i$, $i=\overline{1,n+1}$. From here, get the deflection equations in the local coordinate system $\xi_i\eta_i$ for each $M_{i-1}M_i$, $i=\overline{1,n+1}$ cable segment.

$$\eta_i = \overline{u}_i\left(\xi_i - \frac{\xi_i^2}{\overline{d}_i}\right) + \frac{\left|\overline{y}_i - \overline{y}_{i-1}\right|\xi_i^2}{\overline{d}_i^2} \tag{65}$$

After applying the load, the coordinates of the load points $M_i, i=\overline{1,n}$ is $\overline{x}_i = \sum_{k=1}^{i}\overline{d}_k$ and with $\overline{x}_0 = 0$, $\overline{x}_{n+1} = d$ we have the equation of deflection of the elastic cable OA in the coordinate system Oxy with each segment $\overline{x}_{i-1} \le x \le \overline{x}_i$, $i=\overline{1,n+1}$:

$$y = \begin{cases} \overline{u}_i\left[\left(x - x_{i-1}\right) - \dfrac{\left(x - x_{i-1}\right)^2}{\overline{d}_i}\right] + \dfrac{\left|\overline{y}_i - \overline{y}_{i-1}\right|\left(x - x_{i-1}\right)^2}{\overline{d}_i^2} + \overline{y}_{i-1} & if \quad \overline{y}_i \ge \overline{y}_{i-1} \\ \overline{u}_i\left[\left(x_i - x\right) - \dfrac{\left(x_i - x\right)^2}{\overline{d}_i}\right] + \dfrac{\left|\overline{y}_i - \overline{y}_{i-1}\right|\left(x_i - x\right)^2}{\overline{d}_i^2} + \overline{y}_i & if \quad \overline{y}_i < \overline{y}_{i-1} \end{cases} \tag{66}$$

Tensile force along the cable at two supports are obtained from:

$$\begin{aligned} T_o &= H\sqrt{1+u_1^2} \quad at\ O \\ T_A &= H\sqrt{1+u_{n+1}^2} \quad at\ A \end{aligned} \tag{67}$$

3. Calculation applications

3.1. Elastic cable suspended with point load

The following application is taken as reference to validate different methods to simulate the cables. These cases were analyzed by some researchers such as: O'Brien and Francis (1964), Jayaraman and Knudson (1981), Tibert (1998) and Andreu et al. (2006). The problem is to determine the displacement of the loading point M , when the pre-stressed cable bears its own weight and is subjected to concentrated loads. The cable has a self-weight: $q = 46.12\,N/m$, a cross-sectional area: $F = 5.484\,cm^2$ and an elastic modulus: $E = 13100\,kN/cm^2$. Initial configuration and further information regarding this example are shown in Fig. 4.

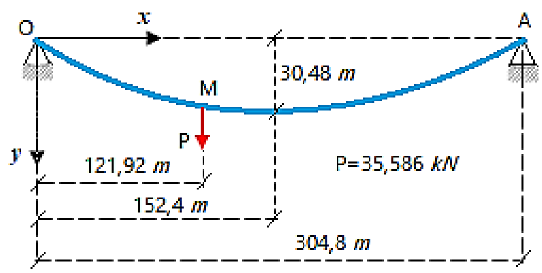

Fig. 4. Cable under self-weight and concentrated load.

Let L_0, u_0 and L, u be the corresponding parameters of the in extension and elastic cable spans in the state of self-load. According to formulas (9) and (14) one gets:

$$L_0 + \Delta L_0 = \frac{d}{2}\left\{\sqrt{1+u_0^2} + \frac{\ln\left(u_0 + \sqrt{1+u_0^2}\right)}{u_0}\right\} + \frac{qd^2}{2EFu_0}\left(1+\frac{u_0^2}{3}\right) = L = \frac{d}{2}\left\{\sqrt{1+u^2} + \frac{\ln\left(u+\sqrt{1+u^2}\right)}{u}\right\} \tag{68}$$

where $d = 304.8m$; $u = \frac{4\times 30.48}{304.8} = 0.4$. Solving equation (68) gets $u_0 = 0.398 \Rightarrow L_0 = 312.666m$. The cable span, when no load is placed at M , has $d_1 = 121.92m$, $h_1 = 29.26m$. When the load is applied at M by using equations (36), (43), (44), (45),(46),(47), the initial values are obtained as: $d_1^{(0)} = 35.586$; $d_2^{(0)} = 182.88$; $u_1^{(0)} = u_2^{(0)} = 0.398$; $h_1^{(0)} = h_2^{(0)} = 29.1164$ via solving the equations with Newton - Raphson method. Get the results: $\overline{h}_1 = \overline{h}_2 = 34.8608m$, $\overline{d}_1 = 121.053m$; displacements at point M in the Ox and Oy directions were compared with the results of different researchers, as shown in Table 3 and on the graph of Fig. 5. It is seen from Table 3 that the results of the current research are in agreement with the previous works and researcher showing the acceptable assumptions and calculations of the cable with parabola type element

Table 3. Comparison of displacements obtained with different works at point M.

Reference	Element type	Displacements (m)	
		Horizontal (Ox)	Vertical (Oy)
O'Brien and Francis (1964)	Elastic straight	-0.845	5.472
Jayaraman and Knudson (1981)	Elastic catenary	-0.859	5.626
Tibert (1999)	Elastic parabola	-0.866	5.601
Andreu (2006)	Elastic catenary	-0.860	5.626
Present work	Elastic parabola	-0.866	5.600

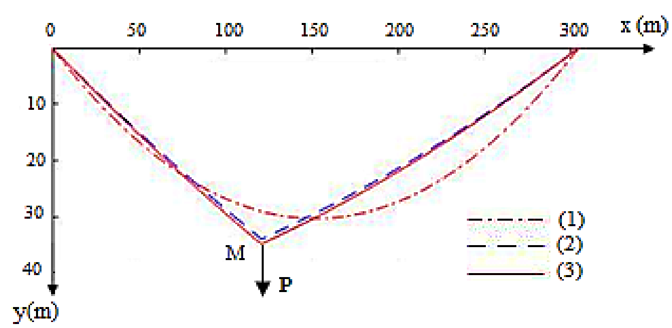

Fig. 5. Image for elastic parabolic cable span under concentrated load.
(1) - The inextensible cable in the state of self-loading; (2) - The inextensible cable in the state of under concentrated load; (3) - The extensible cable in the state of under concentrated load.

3.2. Elastic cable span subjected to many concentrated loads and at the same time are subjected to uniform loads on each segment

The cable span rests on two supports O and A with horizontal distance $d = 200m$ and height difference $h_s = 2m$. Initially, the cable is pulled with a horizontal tension $H = 33{,}333kN$, then concentrated loads $P_1 = 20kN; P_2 = 15kN$ are applied at points M_1, M_2 together with a uniform load of magnitude $q = 90\ kN/m$ on the cable interval M_1M_2 (see Fig. 6). The displacement at points M_1M_2 in vertical and horizontal directions and reactions at two supports O, A were calculated after loading. The cable has a self-weight: $\gamma = 40N/m$, a cross-sectional area: $F = 8\ cm^2$ and an elastic modulus: $E = 13100kN/cm^2$.

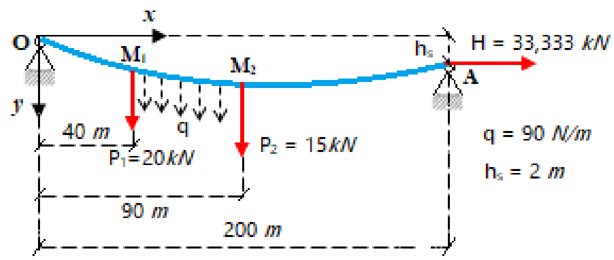

Fig. 6. The elastic cable span simultaneously bears many concentrated loads and uniform loads on each segment.

Let L_0, u_0 and L, u be the corresponding parameters of the in extension and elastic cable spans in the state of self-load. Before the load is applied, the cable span is stretched due to its own weight, with $d = 200\,m$, $h_s = 2\,m$ and horizontal component of cable pulling force $H = 33{,}333\,kN$. According to Eq. (30), get: $u = \dfrac{\gamma.d}{2H} + \dfrac{h_s}{d} = 0{,}13$. The coordinates of the points are M_i: $M_1(40, 4.24)$; $M_2(90, 6.84)$.Using the formulas (22) and (33):

$$L_0 + \Delta L_0 = \frac{d}{2(1-C)}\left\{\sqrt{1+u_0^2} + \frac{\ln\left(u_0+\sqrt{1+u_0^2}\right)}{u_0} - C\sqrt{1+C^2u_0^2} - \frac{\ln\left(Cu_0+\sqrt{1+C^2u_0^2}\right)}{u_0}\right\} + \frac{\gamma d^2}{2E.F.(u_0d-h_s)}\left[d + \frac{du_0^2}{3} + \frac{h_s}{3d}(4h_s - 2u_0d)\right]$$

$$L = \frac{d}{2(1-C)}\left\{\sqrt{1+u^2} + \frac{\ln\left(u+\sqrt{1+u^2}\right)}{u} - C\sqrt{1+C^2u^2} - \frac{\ln\left(Cu+\sqrt{1+C^2u^2}\right)}{u}\right\}, \text{ in which } C = \frac{2h_s}{d} - 1, \ .$$

Solving equation $L_0 + \Delta L_0 = L$ gets $u_0 = 0.13 \Rightarrow L_0 = 200.42\,m$. After applying the load, the set of equations (53), (60), (61), (62) are solved by Newton - Raphson method with initial values: $u_1^{(0)} = 0.121$, $u_2^{(0)} = 0.0766$, $u_3^{(0)} = 0.101$, $y_1^{(0)} = 3.952$, $y_2^{(0)} = 6.394$; $d_1^{(0)} = 40$, $d_2^{(0)} = 50$, $d_3^{(0)} = 110$. The corresponding results are: $\overline{y}_1 = 7.12$; $\overline{y}_2 = 9.43$, $\overline{d}_1 = 39.67\,m$; $\overline{d}_2 = 50.14\,m$. Displacements at point M_i in the Ox and Oy directions is shown in Table 4 and on the graph of Fig. 7. The reactions at O and A in vertical directions: $V_0 = 33{,}152\,kN$; $V_A = 14{,}358\,kN$ and in the horizontal direction is: $H = 180.3\,kN$.

Table 4. Comparison of displacement point M

Point	Displacements (m)	
	Horizontal (Ox)	Vertical (Oy)
M_1	-0,33	2,88
M_2	-0.19	2,59

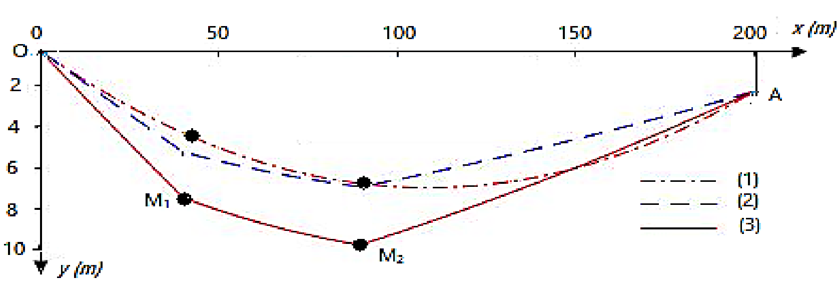

Fig. 7. Image for elastic cable span simultaneously bears many concentrated loads and uniform loads on each segment. (1) - The inextensible cable in the state of self-loading; (2) - The inextensible cable in the state of under concentrated load; (3) - The extensible cable in the state of under concentrated load.

4. Conclusion

With the load evenly distributed on the cable being considered to be evenly distributed in the horizontal direction and the introduction of the dimensionless parameter $u = \frac{4f_c}{d}$, the paper has built a system of non-linear equations, calculated the necessary parameters to get the equation of deflection and tension of the elastic cable resting on two high-displacement bearings, simultaneously bearing many concentrated loads and uniform loads on each cable span. Solving the system of equations is done easily by the Newton-Raphson method. Finally, the example calculation results are compared with the calculated results of other studies as acceptable and demonstrating the validity and accuracy of considering the cable with parabola elastic elements.

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