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Didactic Approaches to Proofs in Mathematics: Difficulties of Proofs

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

The theme of proof in mathematics is often the subject of debate in the educational world. Several different approaches have addressed the problem of learning a proof. In this work we studied the learning of proof in the teaching of mathematics, then we gave analyzes of the difficulties of the pupils to understand its nature and its usefulness. Analyzes of approaches to proof from the teacher's side are provided.

Keywords: Didactic of mathematics; proofs and obstacles in mathematics.

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1 Introduction

The proof, in general according to (Arsac [1], [2]; Balacheff [3]; Daili [4]; Glaeser [5]), is a rigorous and coherent reasoning by which we conclude the need for a proposition from one or more propositions already

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known. It seems to be the best way to lead us to the truth. We can find the proofs in various fields such as: mathematics, physics, economics, sociology ... etc.

In mathematics, a proof is a reasoning which allows, from certain axioms, to establish that an assertion is true. The proofs use constructions based on logic but usually include elements of natural language while avoiding as much as possible the introduction of ambiguities.

So a proof is a chain and each link is nested in others, and the failure of one link causes the whole chain to break.

To build a proof it is not at all sufficient to choose a language and appropriate rules; its construction requires reasoning which can be explained by a particular language and rules.

2 Main Results

What is a difficulty? Are there any difficulties with mathematics proofs? If so, is there a link between reading and proof in mathematics?

Are the difficulties in mathematical proofs of the same order for all age groups? Is there a report of difficulty in mathematical proofs for different genders? How do students and teachers behave when faced with a mathematics proof?

This work represents a complement to the work (Daili [4]) of the author. It gives answers to these questions.

The quests for truth and proof have been part of humanity's history for centuries. Having the right knowledge and proof help us solve current problems, as well as anticipate and solve complex and potential future problems. Most educational programs adhere to the constructivist approach, which emphasizes the importance of proof and true knowledge. According to this approach, new knowledge is constructed on the basis of existing knowledge (Kardag et al [6]; Aksoy and Narli [7]).

Proof is a notoriously difficult mathematical concept for students. Empirical studies have shown that many students emerge from proof-oriented courses such real analysis (Bills and Tall [8]), introduction to proof (Weber [9], [10]; Moore [11]; Knapp [12]; Morali and Ahsen [13]; Sevgi and Kartalci [14]), geometry (Senk [15]), and abstract algebra (Weber [9]) unable to construct anything beyond very trivial proofs. Furthermore, most university students do not know what constitutes a proof (Recio and Godino [16]) and cannot determine whether a purported proof is valid (Selden and Selden [17]).

2.1 The difficulties of proofs in mathematics

Proofs come in all kinds of styles and sizes (Almeida [18]; Velleman [19]; Kiresten and Greefrath [20]). Some are short and effective, like those found in textbooks. Others, which present the very latest research in great detail, are the subject of specific treatment in journals which devote thousands of pages to them (by way of example let us quote two examples: Fermat's theorem and its proof by Wiles; zeros of the zeta function and the results obtained in this direction). Very few people can then get a sense of the full reasoning.

In addition, certain foundations sometimes cause problems. For example, a small number of mathematicians are not satisfied with the method of reasoning by the absurd when it comes to proving the existence. If the proposition that there is no solution to an equation leads to a contradiction, is that sufficient to prove that a solution exists? Critics of this method would argue that this logic is just a sleight of hand and does not tell us how to really build a concrete solution. The Constructivists, who belong to several currents, are the mathematicians who stipulate that the method used in the proof does not give a numerical meaning. They cover with contempt the classical mathematicians who regard the method of recurrence as an essential weapon in the mathematical arsenal. On the other hand, the most traditional mathematicians would say that to proscribe this type of reasoning is to deprive oneself of a very useful tool, and, moreover, that to reject so many results proven by means of this indirect method would leave only the weft of the mathematical tapestry.

2.2 The proof in experimental didactics of mathematics

According to (Glaeser [5]; Pedemonte [21]; Sarr [22]), the theme of proof is often a subject of debate in the educational world. Several different approaches have addressed the problem of learning a proof:

- Epistemological studies to characterize a proof in mathematics;
- Analyzes of students' difficulties to understand its nature and usefulness;
- Analyzes of approaches to proof on the part of the teacher.

2.3 Learning from the proof in mathematics education

There are two opposing views on this subject. The debate is vast and important. We cannot summarize it in a few lines as each author has his conception of this learning. We just want to distinguish two major currents of thought. Some didacticians, like Balacheff ([3], [23]) or Lehman ([24]), refuse to separate learning from proof from work solving problems that give it meaning. The proof should only intervene as a tool and should not be the subject of a specific teaching. Other researchers refuse to equate the proof with a path, natural for some, inconceivable for others. They support the thesis that proof is a notion that can be taught like so many others. The methods vary but some points seem fundamental in this teaching of proof as a technique. The development of the ternary structure (hypotheses, theorem, conclusion) of the basic link of the proof; the status of each statement, to be linked to its language context and not to the content; learning different proof techniques such as proof by absurdity, by contraposition, by disjunction of cases, ... etc.

A teaching of proof is not only possible, but also necessary, without however turning to extremist practices which would make the work of the pupils lose all meaning. And work exercises of the technique of proof based on the deductive step.

2.4 Behavior of students facing to the proof

The opposition between the real objective of a proof and the misinterpretation of this objective by pupils and teachers is the basis of a didactic problem.

Students look for an explanation in a proof. They strive to read a proof as a tool to convince oneself and others. This effort is often requested by teachers without giving students the tools to achieve it. Often students need to do experiments, empirical verifications, after proof because the proof does not convince them. Faced with constructed proof, students often remain skeptical. They fail to understand the guarantee that a given proof in relation to an argument. They prefer narrative arguments that use diagrams because they are closer to their way of expressing justification.

2.5 Difficulties imposed on students

According to Sarr ([22]), among students, we encounter difficulties related to:

2.5.1 The start-up

We situate the problems of starting a proof at the level of:

1) lack of knowledge of the framework and procedures used in the proof;
2) how to use the material at their disposal in the statements, figures and their own knowledge.

2.5.2 In search

In the search for proof, learners are usually not sure where to start. They don't have a research methodology. They do not know how to exploit the clues offered by a statement or a figure.

2.5.3 To the editorial staff

After having looked for a proof, the learners have problems to write it up and to make coherence the ideas which they have brought out. They have problems following the framework of deductive reasoning (hypothesis, theorem, conclusion). They confuse the role and the value of the words or groups of words (conjunctions, adverbs ... etc) used in a proof. To this, we must add the problems related to the ordering of the clauses composing the text of the proof.

2.6 Difficulties imposed on teachers

According to Sarr ([22]), among teachers, the difficulties are of a didactic nature and pedagogical management of large numbers.

2.6.1 Didactic difficulties

2.6.1.1 The lack of benchmarks to give to students

Most teachers provide proofs without explaining the framework, procedures and other elements that make up the proofs. These elements, implicit in the speeches or the answers of the teachers, are sometimes difficult to decipher by the pupil. The latter manage blindly or from vague reminiscences.

2.6.1.2 Lack of relevant activities to offer

Most of the time, the proof is taught in one step without differentiating between the four types of students' difficulties mentioned above. Appropriate activities are not given to the pupils to make them aware of the difficulties and the skills to be developed on each of them.

2.6.2 Difficulties related to class sizes

The problems related to staffing are of the order of:

2.6.2.1 Student-teacher communication

The excess number of students increases the distance between the students and the teacher. It makes movement and proximity difficult which allow a more direct and frequent dialogue between the student and the teacher. It promotes the isolation of some students who need to be "shaken" to invest and encourages them to drown in the mass of strangers.

2.6.2.2 Student-student communication

The excess number of students does not encourage the teacher to develop a continuous debate between the students so as not, as some say, to waste time and promote anarchy. Students do not frequently share their ideas and experiences.

2.6.2.3 Lack of sub-grouping and communication

In general, teachers do not have sub-group and communication techniques that allow students to search together, to share their experiences in a framework full of enthusiasm and fruitfulness.

2.7 Difficulties relating to the construction of proof

According to (Arsac [2]; Barbin [25]; Daili [4]; Pluvinage [26]), the difficulties of proof are numerous and of a very various. Here are some of the most commonly encountered:

- Lexical difficulties: imperfect knowledge of the meaning of terms; this can lead to difficulty in understanding what is required to be proven;

- Difficulties related to the discernment of mathematical objects: not knowing which hypothesis or which proposition to use to prove. This often leads to confusion in concepts;
- Memory-related difficulties: summary or partial recollection of acquired knowledge. This may be due to an imperfection in the acquisition process or a failure in the process of storing mathematical information;
- Difficulties related to reasoning: the construction of a proof calls for innumerable mental operations, complex, which often consist in connecting pieces of knowledge between them, and to adapt them using logical operations to the problem of which one find the solution;
- The difficulty of a proof depends on at least two variables the number of elementary deductions and the number of statements involved (definitions, axioms, or theorem). It is therefore important to know the value of these variables.

3 Conclusions

The proof, in general, is central to any activity of a human being who runs behind the truth.

In particular, proof in mathematics occupies a special place since it represents a fundamental tool which especially characterizes this discipline. It is therefore understandable that it plays an important role in the school curriculum.

It is, therefore, an object of study a priori privileged for the mathematician and all the more so since its introduction is a source of difficulties for many human beings.

Any research on its teaching poses the problem of transmission of its construction, just like any mathematical concept, although the proof is not strictly speaking a concept, but rather a technique.

The importance given to proof is partly attributable to the fact that it separates mathematics from experimental sciences, it also allows to explain, to discover new mathematics, to systematize, to communicate, in addition to offering intellectual challenges its importance in the life of professional mathematicians as well as the various roles and goals for which it is intended confirm and justify that it is indeed an essential characteristic of the mathematical universe.

The proof also has its importance in university education. Indeed, it occupies a prominent place in university-level mathematics courses. In these advanced mathematics courses, taking proof is often the first goal, in addition to being one of the only ways to assess student performance. Proof, at the university level, is a central subject both for its contribution to the mathematical discipline and for the connections it maintains with other fields such as the natural sciences. Proof is at the heart of the mathematician's activity, so it goes without saying that university mathematics devotes a capital place to it.

Its learning requires a change of vision of mathematics, progressing beyond experimental verification and intuition to adopt an approach consistent with a mathematical theory. It is not a question of learning techniques but of knowing how to situate them in the theory because proving it can be learned: it is not the result of a eureka for big head it is the result of a good conduct method.

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Competing Interests

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