Physical Science International Journal
4(9): 1293-1310, 2014
ISSN: 2348-0130

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Effects of Suction and Thermal Radiation on Heat Transfer in a Third Grade Fluid over a Vertical Plate

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/PSIJ/2014/8964
Editor(s):
(1) Christian Brosseau, Distinguished Professor, Department of Physics, Université de Bretagne Occidentale, France.
Reviewers:
(1) Anonymous, Kuvempu University, Shimoga, India.
(2) Anonymous, University of South Carolina Upstate, USA.
(3) Muhammad Sajid, Theoretical Plasma Physics Division, PINSTECH, P.O. Nilore, Islamabad 44000, Pakistan.
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(6) Anonymous, Islamia College University, Peshawar, Pakistan.
Peer review History: http://www.sciencedomain.org/review-history.php?iid=595&id=33&aid=5882

Original Research Article

Received 7th January 2014
Accepted 19th June 2014
Published 25th August 2014

ABSTRACT

Aims: An analysis is presented to investigate the effects of suction and thermal radiation on the unsteady convective flow and heat transfer in a third grade fluid over an infinite vertical plate. The plate is porous to allow for possible wall suction.
Methodology: The governing time-based coupled partial differential equations, subjected to their boundary conditions, are solved numerically by applying an efficient and unconditionally stable Crank-Nicolson finite difference scheme. Numerical calculations are carried out for different values of dimensionless parameters in the problem.
Results: An analysis of the results obtained establishes that the flow field is appreciably

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influenced by suction and viscoelastic parameters.
Conclusion: An increase in the suction parameter is observed to decrease the fluid velocity. The result also shows that the temperature distribution decreases with an increase in the thermal radiation parameter.

Keywords: Suction; thermal radiation; heat transfer; porous plate; third grade fluid.

NOMENCLATURES

Alphabetical Symbols

A_1, A_2, A_3	*Rivlin-Ericksen tensor*
B	*Planck's constant*
C_p	*Specific heat at constant pressure*
Gr	*Thermal Grashof number*
g'	*Acceleration due to gravity*
h	*Step size*
I	*Identity tensor*
i, j	*Grip points along y-direction and t-direction*
n, a, b	*Numerical constants*
$\Pr$	*Prandtl number*
p	*Scalar pressure*
r	*Convergent term*
T'	*Ambient temperature*
T'_w	*Temperature at the wall/plate*
T'_∞	*Free stream concentration*
t', t	*Local time, dimensionless time*
u', v'	*Velocity components in x and y directions*
u	*Fluid velocity*
$U(t)$	*Initial moving velocity*
V_0	*Suction velocity*
x, y	*Coordinate axes*
X_i	*External force in i^{th} direction*

Greek Symbols

α	*Dimensionless second grade viscoelastic parameter*
α_1, α_2	*Second grade viscoelastic material constant*
β	*Dimensionless third grade viscoelastic parameter*
$\beta_1, \beta_2, \beta_3$	*Third grade material constant*

β_T	*Volumetric coefficient of thermal expansion*
θ	*Dimensionless temperature*
η^2	*Absorption coefficient*
λ	*Frequency*
δ	*Radiation absorption coefficient*
κ	*Thermal conductivity*
μ	*Dynamic viscosity*
ν	*Kinematic viscosity*
ω	*Suction parameter*
ρ	*Fluid density*
τ'	*Stress tensor*

Symbols

∇	*Gradient operator*
$\frac{D}{Dt}$	*Material derivative*
$\frac{\partial}{\partial t}$	*Partial time derivative*

Subcripts

w	*Surface condition*
p	*Constant pressure*
∞	*Free stream condition*

1. INTRODUCTION

Non-Newtonian fluids have been a subject of great interest to researchers because of their various industrial and engineering applications. In contrast to the viscous fluids, the non-Newtonian fluids cannot be described by the single constitutive relationship between the stress and the strain rate. In a non-Newtonian fluid, the relation between the shear stress and the strain rate is nonlinear and can even be time-dependent. A third grade non-Newtonian fluid exhibits shear thinning/thickening effects. A third grade fluid is also capable of describing the normal stress differences that is common to second grade fluids. Shear thickening fluids are used in all wheel drive systems that utilize a viscous coupling unit for power transmission. On the other hand, an example of shear thinning fluids is paint. Fundamental areas of applications of these fluids are found in colloidal ceramics processing, plastic manufacture, enhanced oil-recovery systems, lubricants containing polymer additives, biological fluids, food processing, etc. This prevalence in application is due to diverse features of such fluids in nature and industrial applications. Generally, the mathematical problems in third grade non-Newtonian fluids are more complicated because of their high nonlinearity and higher-order nature of differential equations than those in Newtonian fluids. Despite these complexities of the non-Newtonian fluids, scientists and engineers are engaged in the third grade non-Newtonian fluid dynamics.

Erdogan [1] analyzed the flow of a third grade fluid in the vicinity of a plane wall suddenly set in motion. In his observation for a short time, a strong non-Newtonian effect was present in the velocity field. However, for a long time, the velocity field became Newtonian. The problem of peristaltic flow of MHD third order fluid in a planar channel with slip condition was investigated Hayat et al. [2]. The pumping and trapping phenomena were examined in the presence of MHD and slip effects. They derived the solutions under long wavelength and low Reynold's number approximations. Hayat et al. [3] also gave an analytical solution to the flow of a third grade fluid bounded by two parallel porous plates using homotopy analysis method (HAM). They made a comparison with the exact numerical solution for various values of the physical parameters.

Sajid and Hayat [4] also proposed a non-similar series solution to two-dimensional boundary layer flow of a third grade fluid over a stretching sheet. Sajid et al. [5] considered heat transfer characteristics in an electrically conducting third grade fluid. They employed non-similar analytic solution for MHD flow and heat transfer in a third order fluid over a stretching sheet. The series solution to the unsteady boundary layer flow of a third grade fluid was developed by Abbasbandy and Hayat [6]. Siddiqui et al. [7] investigated the heat transfer flow problem of a third grade fluid between two heated parallel plates for the constant viscosity model. Three flow problems of Couette flow, plane Poiseuille flow and plane Couette-Poiseuille flow were examined by them. They employed the homotopy perturbation technique to obtain their results. Hayat et al. [8] studied exact solutions of the thin film flow problem for a third grade fluid on an inclined plane. They compared their results with those of Siddiqui et al. [7] and concluded that their solutions were valid for large values of the material parameter.

Analyses of flow scenario on porous surfaces find applications in extrusion of plastic, petroleum engineering, contamination technologies, biotechnology and non-Newtonian chemical materials processing. One can refer to some useful works of Hayat and his co-workers [9-15], regarding the flow and heat transfer in a third grade fluid with different geometries and diverse physical characteristics. Sahoo [16] numerically analyzed the problem of Heimenz flow and heat transfer of a third grade fluid using the finite difference technique with Richardson's extrapolation. Also, Ellahi et al. [17] presented the heat transfer analysis on the laminar flow of an incompressible third grade fluid through a porous flat channel. They provided analytical solution for temperature distribution for various values of the controlling parameters, compared the results obtained with the numerical solution and the comparison showed the fact that the accuracy was remarkable. Sahoo and Do [18] investigated the effects of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet. They employed an effective second order numerical scheme of finite difference technique with Broyden's method and addressed the issue of paucity of boundary conditions involved.

Furthermore, Ellahi and Hamed [19] numerically made an interesting study for the steady non-Newtonian flows with heat transfer, MHD and nonslip effects. Nayak et al. [20] studied the flow and heat transfer of a third grade fluid past a porous vertical plate. They obtained solutions through a numerical approach. Sibanda et al. [21] proposed the problem of heat transfer flow of a third grade fluid between parallel plates using the spectral homotopy analysis method. Explicit analytical expressions for the non-linear momentum and energy equations were solved using the homotopy perturbation method. Recently, Hayat et al. [22] carried out an analysis for the characteristics of melting heat transfer in the boundary layer flow of third grade fluid in a region of stagnation point past a stretching sheet. They developed the series solutions by homotopy analysis method and compared their results with the previous studies. Baoku et al. [23] reported the solution to the problem of MHD partial slip flow, heat and mass transfer of a viscoelastic third grade fluid over an insulated porous plate embedded in a porous medium. They presented numerical experiments of midpoint scheme with Richardson's extrapolation to solve the governing coupled highly nonlinear ordinary differential equations of momentum, energy and concentration. They also showed the effects of the various physical parameters on the velocity, temperature and concentration distributions.

The influence of thermal radiation on flow and heat transfer processes is of paramount interest in physics and engineering, particularly in the design of many advanced energy conversion systems operating at high temperature (Seddeek [24]), in power generator system, cooling of nuclear reactors, high temperature plasma and in controlling heat transfer in polymer processing industry where quality of the final products largely depends on the heat controlling factors. Thermal radiation, within such energy conversion systems, occurs because of the emission by the hot walls and working fluid. Plumb et al. [25] studied the effect of horizontal cross-flow and radiation on natural convection from vertical heated surface in saturated porous media. Rosseland diffusion approximation was utilized for the convective flow with radiation. Hossain and Takhar [26], Takhar et al. [27], Hossain et al. [28] extensively investigated the effect of radiation on heat transfer problems.

Mansour [29] also analyzed the combined forced-convective flow over a flat plate immersed in porous medium of variable viscosity. Sajid and Hayat [30] examined the problem of radiation effects on the flow over an exponentially stretching sheet and solved the problem analytically using the homotopy analysis method. The numerical solution for the problem was then provided by Bidin and Nazar [31]. Anand et al. [32] critically observed radiation effects on an unsteady MHD free convective flow past a vertical porous plate in the presence of soret effect. Seethamahalakshmi et al. [33] investigated the unsteady MHD free convective flow and mass transfer near a moving vertical plate in the presence of thermal radiation. Some other research works that have been carried out on this area are those of Makinde et al. [34], Srinivas and Muthuraj [35] and Singh et al. [36]. Lastly, Baoku et al. [37] examined the influence of thermal radiation on a transient magnetohydrodynamic Couette flow of a high Prandtl number fluid with temperature-dependent viscosity through a porous medium. They employed an implicit finite difference scheme of Crank-Nicolson type to investigate the effects of pertinent flow parameters.

The aim of this study is to investigate the suction and thermal radiation effects in a thermodynamically compatible viscoelastic third grade fluid on unsteady flow and convective heat transfer over an infinite porous plate which is set in motion with an oscillating temperature applied to the plate. The governing coupled nonlinear partial differential equations with sufficient initial and boundary conditions are solved by employing Crank-Nicolson finite difference scheme with modified Newton's method. The present problem with radiative heat flux has not been considered in the scientific literature to the best knowledge of the author, despite its important applications in industry and engineering.

2. MATHEMATICAL ANALYSIS

Consider the transient flow and heat transfer of an incompressible fluid of a third grade past infinite porous plate. The x' - axis is taken along the plate vertically upwards and y' - axis is normal to it as shown in Fig. 1. The plate is suddenly set in motion in its own plane with a velocity U(t). An oscillating temperature is assumed to be applied on the plate in the presence of thermal radiation. It is also presumed that the plate is infinitely long. Thus, the physical variables are functions of y' and t' only. Hence, from the continuity equation, the velocity field is described as:

$$u' = u'(y', t'),\ v' = -V_0 \tag{1}$$

Where u' and v' are the velocities of the fluid along x' and y' axes respectively and $V_0 \succ 0$ indicates suction velocity.

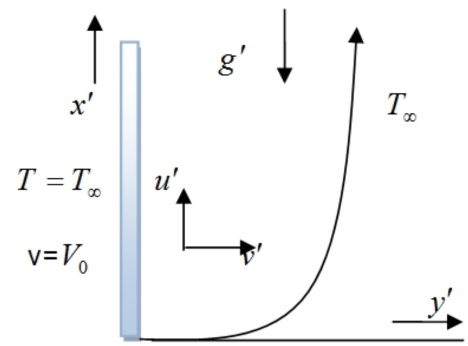

Fig. 1. Schematic diagram and coordinate system

2.1 Flow Analysis

The constitutive equation of an incompressible third grade fluid as given by Coleman and Noll [38] is:

$$\tau' = -pI + \sum_{i=1}^{3} S_i \tag{2}$$

Where $S_1 = \mu A_1$, $S_2 = \alpha_1 A_2 + \alpha_2 A_1^2$ and $S_3 = \beta_1 A_3 + \beta_2 (A_2 A_1 + A_1 A_2) + \beta_3 (trA_2) A_1$. $\mu, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ being material constants, τ' the stress-tensor, p the pressure, I the identity tensor and A_n represents the kinematical tensors defined by, $A_0 = I$, $A_1 = \nabla u + (\nabla u)^T$, $A_{n+1} = \left(\frac{\partial}{\partial t} + u \cdot \nabla\right) A_n + \nabla u \cdot A_n + (\nabla u \cdot A_n)^T$, $n = 1, 2$.

Where u is the velocity and t is the time. A detailed thermodynamic analysis of the model, represented by (2) is given by Fosdick and Rajagopal [39]. It was shown that if all the motions of the fluid are to be compatible with thermodynamics in the sense that these motions meet the Clausius-Duhem inequality and if it is assumed that the specific Helmholtz free energy is a minimum when the fluid is locally at rest, then

$\mu_1 \geq 0,\ \alpha_1 \geq 0$, $|\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}$, $\beta_1 = \beta_2 = 0$, $\beta_3 \geq 0$ and

$$\tau' = -pI + \mu_1 A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 \left(trA_1^2\right) A_1 \tag{3}$$

The stress components (2) by virtue of equation (1) are:

$$\tau_{x'x'} = -p + \alpha_2 \left(\frac{\partial u'}{\partial y'}\right)^2 + 2\beta_2 \frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'},$$

$$\tau_{y'y'} = -p + \left(2\alpha_1 + \alpha_2\right)\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(6\beta_1 + 2\beta_2\right)\frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'},$$

$$\tau_{z'z'} = -p, \tag{4}$$

$$\tau_{x'y'} = \mu \frac{\partial u'}{\partial y'} - \alpha_1 V_0 \frac{\partial^2 u'}{\partial y'^2} + \alpha_1 \frac{\partial^2 u'}{\partial y' \partial t'} + 2\left(\beta_2 + \beta_3\right)\left(\frac{\partial u'}{\partial y'}\right)^3 + \beta_1 \left(\frac{\partial^3 u'}{\partial y' \partial t'^2}\right),$$

$$\tau_{x'z'} = \tau_{z'y'} = 0$$

Where $\tau_{x'y'} = \tau_{y'x'}$, $\tau_{x'z'} = \tau_{z'x'}$, $\tau_{y'z'} = \tau_{z'y'}$

Inserting the stress components using (3) and velocity given by (1) in the equation of motion:

$$\rho \frac{Dv_i}{Dt} = -\tau_i + \rho X_i + \tau_{ij,j} \tag{5}$$

Where $\frac{D}{Dt}$ denotes the material derivative and ρX_i is the external force per unit mass in i^{th} direction, the governing equation of free convective flow field under the physical conditions of the problem is obtained as:

$$\rho\left(\frac{\partial u'}{\partial t'} - V_0 \frac{\partial u'}{\partial y'}\right) = \mu \frac{\partial^2 u'}{\partial y'^2} + \alpha_1 \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} - V_0 \frac{\partial^3 u'}{\partial y'^3}\right) + 6\beta_3 \left(\frac{\partial u'}{\partial y'}\right)^2 \frac{\partial^2 u'}{\partial y'^2} + \rho \beta_T g\left(T' - T'_\infty\right) \tag{6}$$

2.2 Heat Transfer Analysis

Neglecting viscous dissipation, the heat transport equation is obtained as:

$$\rho C_p \frac{DT}{Dt} = K \nabla^2 T - \nabla q_r \tag{7}$$

Assuming the conditions of optically thin environment, the radiative heat flux, $\partial q' / \partial y'$ in the energy equation takes the form Takhar, et al. [27]: $\dfrac{\partial q'}{\partial y'} = 4\eta^2 \left(T' - T'_\infty\right)$ where $\eta^2 = \int_0^\infty \left(\delta\lambda \dfrac{\partial B}{\partial T'}\right)$; η^2, δ, λ and B are respectively absorption coefficient, radiation absorption coefficient, frequency and Planck's constant.

The governing equation of temperature flow field is obtained as:

$$\rho C_p \left(\frac{\partial T'}{\partial t'} - V_0 \frac{\partial T'}{\partial y'}\right) = K \frac{\partial^2 T'}{\partial y'^2} - 4\eta^2 \left(T' - T'_\infty\right) \tag{8}$$

In the energy equation (7), the term representing viscous and joule dissipation are assumed to be neglected as they are really very small in slow motion free convective flows.

Initial and boundary conditions for the flow field are:

$$t' \le 0 : y = 0, u' = 0, T' = 0,$$

$$t' \succ 0 : u' = U(t') = \frac{U^{2n+1}}{\nu^n} \ell^{a't'} t'^n, \; when \;\; y' = 0$$

$$T' = T'_\infty + \left(T'_w - T'_\infty\right) \cos b't', \; when \;\; y' = 0 \tag{9}$$

$$u' = 0, \; \frac{\partial u'}{\partial y'} = 0, \; when \;\; y' \to \infty$$

$$T' \to T'_\infty, \; when \;\; y' \to \infty$$

For computing the solution, choosing $U(t') = \dfrac{U^{2n+1}}{\nu^n} \ell^{a't'} t'^n$ where $\nu = \dfrac{\mu}{\rho}$ is the kinematic viscosity and introducing the following dimensionless variables:

$$u = \frac{u'}{U}, \; y = \frac{y'U}{\nu}, \; a = \frac{a'\nu}{U^2}, \; t = \frac{t'U^2}{\nu}, \; b = \frac{b'\nu}{U^2}, \; \omega = \frac{V_0}{U} \tag{10}$$

Equation (6), with the similarity transformation for scaling temperature in heat transfer analysis $\left(T' = \theta\left(T'_w - T'_\infty\right) + T'_\infty\right)$, becomes:

$$\frac{U^3}{\nu}\frac{\partial u}{\partial t}-\frac{V_0U^2}{\nu}\frac{\partial u}{\partial y}=\frac{\mu U^3}{\rho\nu^2}\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\frac{U^5}{\nu^3}\frac{\partial^3 u}{\partial y^2\partial t}-\frac{\alpha_1}{\rho}\frac{V_0U^4}{\nu^3}\frac{\partial^3 u}{\partial y^3}$$
$$+6\frac{\beta_3}{\rho}\frac{U^7}{\nu^4}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+\beta_T g'\left[\theta\left(T'_w-T'_\infty\right)+T'_\infty-T'_\infty\right]$$

$$\Rightarrow\begin{array}{l}\dfrac{\partial u}{\partial t}-\dfrac{V_0}{U}\dfrac{\partial u}{\partial y}=\dfrac{\mu}{\rho\nu}\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\alpha_1}{\rho}\dfrac{U^2}{\nu^2}\dfrac{\partial^3 u}{\partial y^2\partial t}-\dfrac{\alpha_1}{\rho}\dfrac{U^2}{\nu^2}\dfrac{V_0}{U}\dfrac{\partial^3 u}{\partial y^3}\\ +6\dfrac{\beta_3}{\rho}\dfrac{U^4}{\nu^3}\left(\dfrac{\partial u}{\partial y}\right)^2\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\nu}{U^3}\beta_T g'\left(T'_w-T'_\infty\right)\theta\end{array}\tag{11}$$

Similarly, equation (8) becomes:

$$\rho C_p\frac{U^2}{\nu}\left(T'_w-T'_\infty\right)\frac{\partial\theta}{\partial t}-\frac{V_0U}{\nu}\rho C_p\left(T'_w-T'_\infty\right)\frac{\partial\theta}{\partial y}$$
$$=\frac{\kappa U^2}{\nu^2}\left(T'_w-T'_\infty\right)\frac{\partial^2\theta}{\partial y^2}-4\left(T'_w-T'_\infty\right)\xi^2\theta$$
$$\Rightarrow\frac{\partial\theta}{\partial t}-\frac{V_0}{U}\frac{\partial\theta}{\partial y}=\frac{\kappa}{\rho C_p\nu}\frac{\partial^2\theta}{\partial y^2}-\frac{4\xi^2\nu}{\rho C_p U^2}\theta\tag{12}$$

Using the above equations (11-12), equations (6) and (8) reduce to:

$$\frac{\partial u}{\partial t}-\omega\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}-\omega\alpha\frac{\partial^3 u}{\partial y'^3}+\beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+Gr\theta\tag{13}$$

$$\frac{\partial\theta}{\partial t}=\omega\frac{\partial\theta}{\partial y}+\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-R_d\theta\tag{14}$$

where $\alpha=\dfrac{\alpha_1U^2}{\rho\nu^2}$, $\beta=\dfrac{6\beta_3U^4}{\rho\nu^3}$, $Gr=\dfrac{\beta g\nu\left(T'_w-T'_\infty\right)}{U^3}$, $\Pr=\dfrac{\nu\rho C_p}{K}$, $R_d=\dfrac{4\eta^2\nu}{\rho C_p U^2}$

Using the dimensionless variables in (10), the initial and boundary conditions become:

$$\begin{array}{l}t\le 0: y=0,\ u=0,\ \theta=0;\\ t\succ 0: u=U(t)=\ell^{at}t^n,\ when\ y=0;\\ \theta=\cos bt,\ when\ \ y=0;\end{array}\tag{15}$$

$$u=0,\frac{\partial u}{\partial y}=0,\ \theta=0;\ when\ y\to\infty.$$

3. NUMERICAL SIMULATION

The governing nonlinear coupled partial differential equations (13) and (14) with the initial and boundary conditions (15) are solved by employing Crank-Nicolson finite difference scheme which has been discussed by Ganesan and Palani [40], Conte and De Boor [41], Jain [42] and Baoku et al. [37]. Therefore, the governing equations based on the unsteady state conditions are discretized using the method. The finite difference equations corresponding to these governing equations are given as:

$$
\begin{aligned}
u_{i,j+1}-u_{i,j} &= \left(\frac{\omega r h}{4}+\frac{r}{2}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i+1,j+1}+\left(\frac{r}{2}-\frac{\omega r h}{4}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i-1,j+1} \\
&+\left(\frac{\omega r h}{4}+\frac{r}{2}\right)u_{i+1,j}+\left(\frac{r}{2}-\frac{\omega r h}{4}+\frac{\alpha}{h^2}\right)u_{i-1,j}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i,j+1}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i,j} \\
&+\frac{\alpha}{h^2}u_{i+1,j}-\frac{\omega\alpha r}{2h}\left(-u_{i-2,j+1}+u_{i+2,j+1}+u_{i-2,j}-2u_{i-1,j}+2u_{i+1,j}-u_{i+2,j}\right) \\
&+\frac{\beta r}{32h}\begin{bmatrix}\left(u_{i+1,j+1}\right)^2+\left(u_{i-1,j+1}\right)^2+\left(u_{i+1,j}\right)^2+\left(u_{i-1,j}\right)^2-2u_{i+1,j+1}u_{i-1,j-1}+2u_{i+1,j+1}u_{i+1,j} \\ -2u_{i+1,j+1}u_{i-1,j}-2u_{i+1,j}u_{i-1,j-1}+2u_{i-1,j}u_{i-1,j-1}-2u_{i+1,j}u_{i-1,j}\end{bmatrix} \\
&\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)+r\,Grh^2\theta_{i,j}
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\theta_{i,j+1} &= \left(\frac{\omega r h}{2}+\frac{r}{\Pr}\right)\theta_{i+1,j+1}+\left(\frac{r}{\Pr}-\frac{\omega r h}{2}\right)\theta_{i-1,j+1}-\left(\frac{2r}{\Pr}+\frac{R_d r h^2}{2}\right)\theta_{i,j+1} \\
&+\left(1-\frac{R_d r h^2}{2}\right)\theta_{i,j}
\end{aligned}
\tag{17}
$$

Where i designates the grip point along y-direction, j along t-direction and $r = \Delta t / h^2$.

The numerical finite difference method of Crank-Nicolson type does not restrict the value of r to be chosen. Hence, the equations of motion and energy are reduced to system of algebraic nonlinear coupled-equations. The mesh size h is 0.05 with time step $t = 0.1$. The values of $u(y,t)$ and $\theta(y,t)$ are known at all grip points when $t = 0$ from the initial conditions. Modified Newton's iterative technique is then used to solve the system of nonlinear algebraic equations. Computations are carried out by moving along y-direction. After computing values corresponding to each i at a time level, the values at the next time level are determined in similar manner.

The implicit nature of Crank-Nicolson method is unconditionally stable and has local truncation error $O\left[(\Delta t)^2, h^2\right]$ which tends to zero as Δt and h^2 tend to zero. There is no drawback of conditionally stability from one level to the next. The implicit method gives stable solutions and requires iterative procedure which was done at step forward in time because this problem is an initial-boundary value problem with a finite number of spatial grip points. Though, the corresponding difference equations do not automatically guarantee the convergence of the mesh $h \to 0$. To achieve maximum numerical efficiency, the tridiagonal procedure was used to solve the two point conditions for (14) and four point conditions for (13). The above procedure was transformed into Maple code as described by Heck [43]. The convergence of the process was quite satisfactory and the numerical stability of the method was guaranteed by the implicit nature of the scheme. Hence, the scheme is consistent; stability and consistency ensure convergence.

4. DISCUSSION OF RESULTS

The investigation focuses on the flow fields when a vertically upward porous plate suddenly starts moving with a velocity in its own plane and temperature field, assumed to be oscillating, is applied to the plate in the presence of suction and thermal radiation. The governing equations of the flow and temperature fields are solved using Crank-Nicolson implicit finite difference scheme with modified Newton's method and approximate solutions are obtained for the velocity and temperature profiles. The effects of the pertinent parameters on the flow and temperature fields are analyzed and discussed with the help of velocity profiles (Figs. 2 - 6) and temperature profiles (Figs. 7-9).

4.1 Velocity Profiles

The effects of various parameters on the velocity field are investigated through simulations using the method above and results are shown as graphs for two major cases; $n = 0.8$, i.e. when the plate starts moving with variable accceleration and $n = 1$, i.e. when the plate starts with a constant acceleration. Fig. 2 analyzes the influence of suction parameter ω for cases $n = 0.8$ and $n = 1$. It is observed that an increase in the suction parameter ω decreases the fluid velocity at any point of the fluid, and higher velocity profile is attained when $n = 0.8$. This implies that suction which is the removal of fluid from the domain via the porous plate can be used to control the fluid dynamics. Thus, suction enhances adherence of the fluid to the plate which in turn retards the flow. This observation is in conformity with that of Beg et al. [44]. Figs. 3 and 4 depict the effect of viscoelastic parameters α and β on the velocity field. It is observed that as the viscoelastic parameter α increases, the velocity field increases. The influence of β on the velocity profile is noticeable when there is small increment in time interval. An increase in β corresponds to a slight growing in the velocity profiles for both cases of constant and variable accelerations. Hence, the viscoelastic parameters are seen to boost the velocities to the plate surface with suction present. This observation agrees with that reported by Beg et al. [44].

As the free convection current exists by virtue of temperature difference $\left(T' - T'_\infty\right)$, the Grashof number Gr can realistically take any real number when $0 \le b \le \frac{\pi}{2}$. $Gr \succ 0$ corresponds to cooling of the plate while $Gr \prec 0$ corresponds to heating of the plate due to free convection current. Therefore, both positive and negative values have been chosen for

Grashof number. An increase in Grashof number Gr increases the fluid velocity near the plate when the plate is being heated for both $n = 0.8$ and $n = 1$ in Fig. 5. However, when the plate is being cooled, the fluid velocity decreases as the Grashof number increases. Also, Fig. 6 shows that an increase in Prandtl number $\Pr$ increases the fluid velocity in both cases of constant and variable accelerations. It is worth mentioning that these findings on Gr and $\Pr$ are in consonant with those of Sahoo [45].

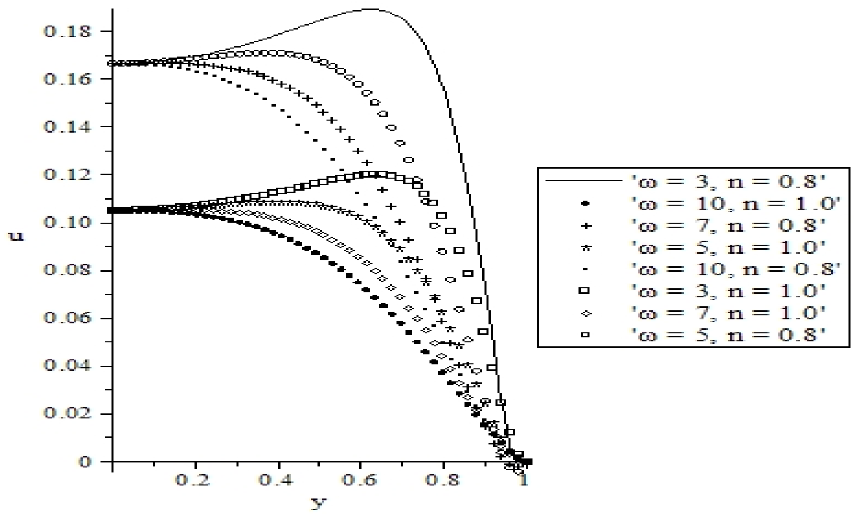

Fig. 2. Effect of ω on velocity field when $n = 0.8$ and $n = 1$ with $\alpha = 1$, $\beta = 1$, $Gr = 10$, $\Pr = 10$

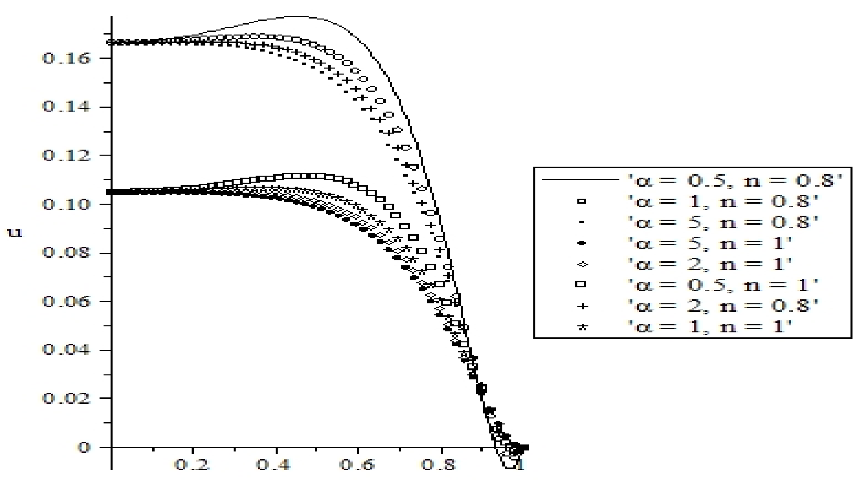

Fig. 3. Effect of α on velocity field when $n = 0.8$ and $n = 1$ with $\omega = 5$, $\beta = 1$, $Gr = 10$, $\Pr = 10$

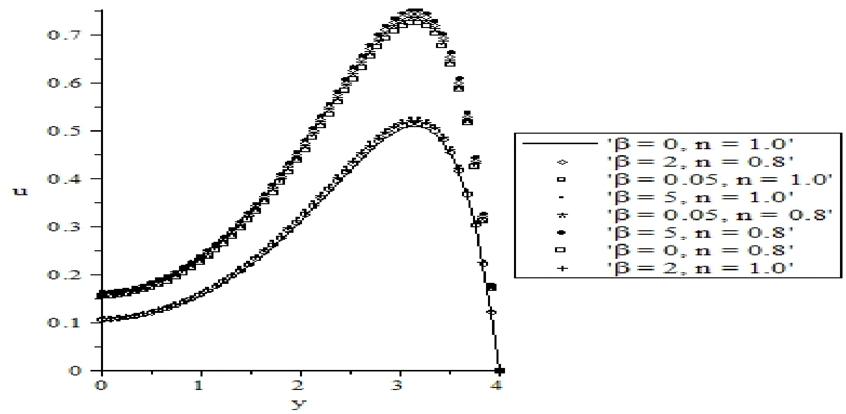

Fig. 4. Effect of β on velocity field when $n = 0.8$ and $n = 1$ with $\alpha = 1, \omega = 5$, $Gr = 10, \Pr = 10$

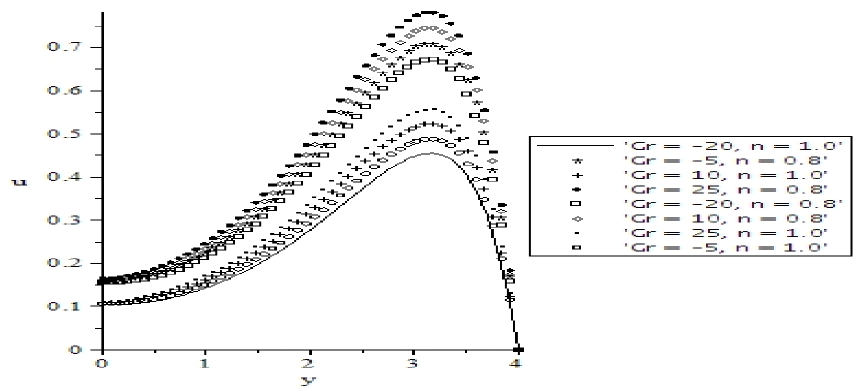

Fig. 5. Effect of Gr on velocity field when $n = 0.8$ and $n = 1$ with $\alpha = 1$, $\beta = 1$, $\omega = 10$, $\Pr = 10$

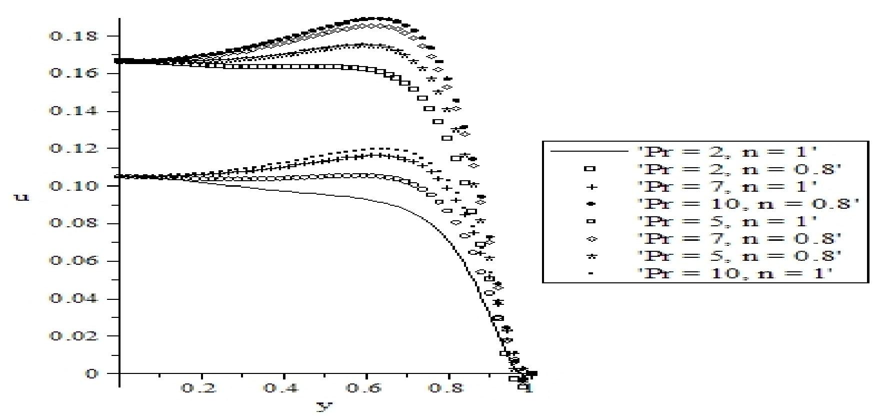

Fig. 6. Effect of $\Pr$ on velocity field when $n = 0.8$ and $n = 1$ with $\alpha = 1$, $\beta = 1$, $Gr = 10$, $\omega = 5$

4.2 Temperature Fields

The temperature of the flow field suffers a substantial change with the variation of the flow parameters such as suction parameter ω, Prandtl number $\Pr$ and thermal radiation parameter R_d. These variations are shown in Figs. 7-9. Fig. 7 expresses the influence of suction parameter ω on the temperature field. A growing ω is found to decrease the temperature of the flow field at all points in the domain. Similarly, it is observed in Fig. 8 that the effect of increasing $\Pr$ reduces the temperature field. This observation on $\Pr$ quite conforms to that reported by Sahoo [45]. Lastly, it is evident from Fig. 9 that at lower value of R_d, there is little or no influence of R_d on the temperature profile whereas at higher value of R_d, the effect of R_d on temperature distribution is noticeable. Hence, the consequence of increasing R_d has the influence of decreasing the temperature of the flow field.

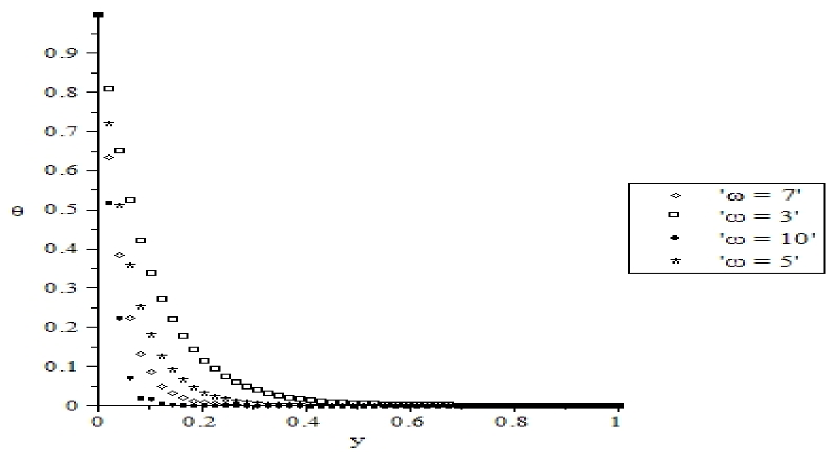

Fig. 7. Effect of ω on temperature field when $R_d = 5$ and $\Pr = 10$

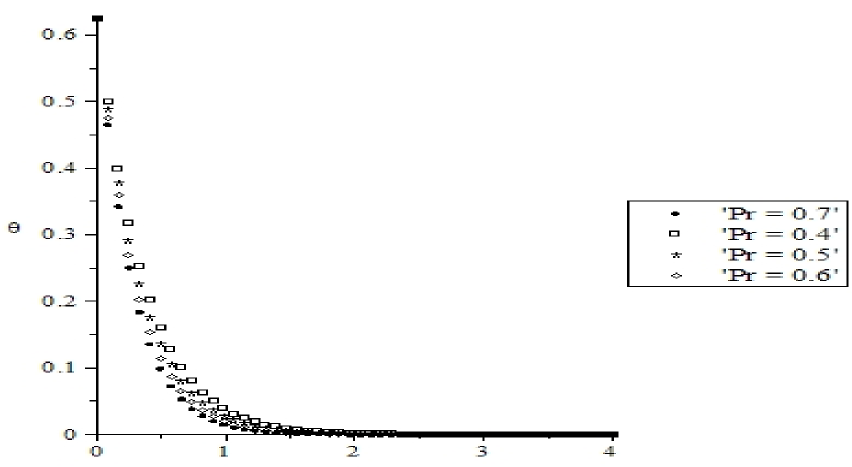

Fig. 8. Effect of $\Pr$ on temperature field when $R_d = 5$ and $\omega = 5$

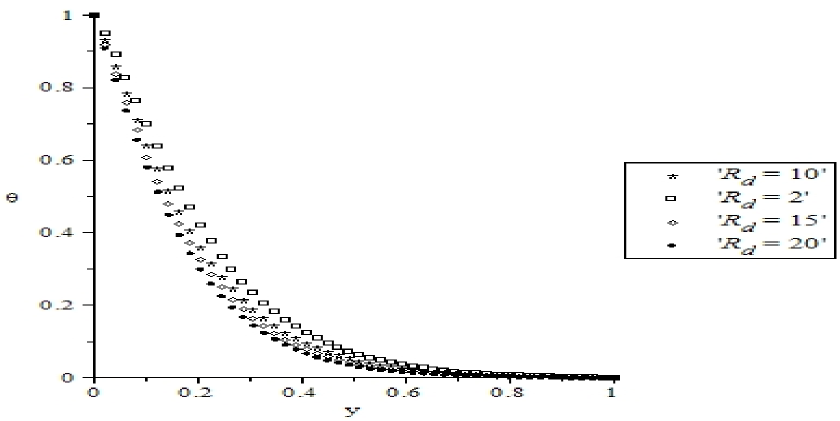

Fig. 9. Effect of R_d on temperature field when $\omega = 5$ and $\Pr = 10$

5. CONCLUSION

In this study, the influence of suction and thermal radiation is investigated on the transient flow and heat transfer of a third grade viscoelastic fluid through a vertical porous plate. An implicit finite difference numerical scheme of Crank-Nicolson type is employed to discretize the system of coupled partial diffential equations and modified Newton's method is used to solve the system of algebraic nonlinear equations obtained after discretization. The above scheme is transformed into the Maple code to simulate the solutions of the problem. This solution procedure is valid for all values of viscoelastic parameters unlike perturbation and power series methods that are valid for small values of viscoelastic parameters.

Therefore, results of physical interest on the velocity and temperature distribution of the flow field are summarized below:

- The fluid velocity increases when the value of the second grade viscoelastic parameter α increases. Also, it increases with an increase in the third grade viscoelastic parameter β for small increment in the time interval.
- The suction parameter ω has the influence of reducing the velocity and temperature field.
- As the Prandtl number increases, it also increases the velocity field but it reduces the temperature distribution of the flow field.
- The fluid velocity increases when the plate is being heated and decreases when the plate is being cooled with higher velocity profile noticeable when the plate starts moving with variable acceleration.
- The effect of increasing the thermal radiation parameter R_d decreases the temperature distribution of the flow field.

ACKNOWLEDGEMENTS

The author is grateful to the editor and reviewers for their constructive comments and valuable suggestions to improve this research paper. Also, the author shows his appreciation to Late Prof. R.O. Ayeni, Department of Pure and Applied Mathematics, Ladoke

Akintola University of Technology Ogbomoso Nigeria, for his contribution in the author's academic endeavours.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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