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Ruin Probabilities in a Discrete Semi-Markov Risk Model with Random Dividends to Shareholders and Policyholders

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

The discrete semi-Markov risk model is modified by the inclusion of dividends paying to shareholders and policyholders. When surplus is no less than the thresholds a_1 and a_2, the company randomly pays dividends to shareholders and policyholders with probabilities q_1,q_2 respectively. Recursive formulae for ruin probabilities are derived. Finally, a numerical example is given to illustrate the effect of the related parameters on the ruin probabilities.

Keywords: Discrete semi-Markov risk model; random dividends; ruin probability.

1 Introduction

During the past decades, the concept of a discrete semi-Markov risk model was investigated quite intensively. The classical semi-Markov risk model is a discrete risk process with the following features. Let $(J_n, n \in \mathbb{N})$ be a homogeneous, irreducible and aperiodic Markov chain with finite

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state space $M = \{1, \dots, m\}$ $(m \in \mathbb{N}^+)$. Its one-step transition probability matrix $\mathbf{P} = (p_{ij})_{i,j\in M}$ is given by

$$p_{ij} = \mathbb{P}(J_n = j | J_{n-1} = i, J_k, k \le n-1),$$

with a unique stationary distribution $\pi = (\pi_1, \dots, \pi_m)$. The surplus at time t is

$$U(t) = u + t - \sum_{i=1}^{t} Y_i, \quad t \in \mathbb{N}^+, \tag{1.1}$$

where $U(0) = u$ is the initial surplus. The total individual claim amounts is Y_i in the i-th period. The premium received in each period is 1. The distribution of $\{Y_n, n \in \mathbb{N}^+\}$ is a sequence of non-negative integer random variables, conditionally independent given the Markov chain $(J_n, n \in \mathbb{N})$. Suppose that (J_t, Y_t) depends on $\{J_k, X_k; k \le t-1\}$ only through J_{t-1}. Define

$$g_{ij}(l) = \mathbb{P}(Y_t = l, J_t = j | J_{t-1} = i, J_k, Y_k, k \le t-1), \quad l \in \mathbb{N}, \tag{1.2}$$

and

$$g_i(k) = \sum_{j=1}^{m} g_{ij}(k), \mu_{ij} = \sum_{k=0}^{\infty} k g_{ij}(k) < \infty, \ \mu_i = \sum_{j=1}^{m} \mu_{ij}, \quad i \in M.$$

The risk model (1.1) was first proposed by Janssen and Reinhard [1][2] and has been further studied by many authors during the last few years. See, for example, Cheung and Landriault [3], Chen et al.[4][5]. Recently, the theory of semi-Markov risk model has developed in a variety of directions. The randomized dividend strategy was studied by Tan and Yang [6] for the compound binomial model. Based on the discrete semi-Markov risk model, Chen et al. incorporated randomized dividends into the discrete semi-Markov risk model. Under the dividend payment strategy, when the surplus is no less than the threshold, the insurer pays a dividend of 1 with probability $1 - \alpha$ at the beginning of the period time. More results about discrete semi-Markov risk model about random dividends can be found in [7][8].

In this paper, motivated by the work of Tan and Yang [6] and Chen et al.[4], we modify the model by paying dividends to shareholders and policyholders and giving two thresholds, which makes insurance company randomly decide to pay dividends to shareholders and policyholders according to the thresholds. Then we derive the recursion formulas for the survival probabilities and the ruin probability.

The rest of the paper is organized as follows. In section 2, the model is introduced. In section 3, we give the recursion formulas for the survival probability. Lastly, a numerical example is also given to illustrate the effect of parameters on the ruin probability.

2 The Risk Model

Now the surplus process (1.1) is modified as follows: Give two thresholds a_i, $i = 1, 2$, where a_1, a_2 are fixed non-negative integers and $a_2 > a_1$. The insurance company pays dividends for shareholders and policyholders according to the threshold. Assuming the current surplus is greater or equal to the threshold, the company will pay a dividend of 1 with probability q_i, $i = 1, 2$. Then the modified surplus at the end of the $t-$th period is given by

$$U(t) = u + t - \sum_{i=1}^{t} Y_i - \sum_{i=1}^{t} [\eta_i^{(1)} 1_{(u_{i-1} \ge a_1)} + \eta_i^{(2)} 1_{(u_{i-1} \ge a_2)}], \quad t \in \mathbb{N}^+, \tag{2.1}$$

where claim sizes $\{Y_i, i \in \mathbb{N}^+\}$ depend on the Markov chain $(J_n, n \in \mathbb{N})$ through the relationship(1.2). $\{\eta_i^{(j)}, i \in \mathbb{N}\}$,$j = 1, 2$ are i.i.d. $1_{(A)}$ is the indicator function of a set A. Bernoulli sequences with

$P(\eta_i^{(j)} = 0) = p_j = 1 - q_j > 0, j = 1, 2$, respectively. Define $\tau = \inf\{t \in \mathbb{N} : U_t < 0\}$ to be the ruin time. Define the ultimate ruin probability $\psi_i(u) = P(\tau < \infty | U_0 = u, J_0 = i), i \in M, u \in \mathbb{N}$. Let the corresponding survival probability be $\phi_i(u) = 1 - \psi_i(u)$. In this paper, we always assume that the positive security loading condition holds, i.e. $\sum_{i=1}^m \pi_i\mu_i < 1 - q_1 - q_2$.

3 Recursive Formulae for Ruin Probabilities

In this section, we derive the recursive formulas of the survival probability. Consider the special case of $m = 2$ and the dividend in the first time period $[0, 1]$. We separate the three possible cases as following:

$$\phi_i(u) = \sum_{j=1}^{2}\sum_{k=0}^{u+1} g_{ij}(k)\phi_j(u+1-k), \quad 0 \le u < a_1, \tag{3.1}$$

$$\phi_i(u) = p_1\sum_{j=1}^{2}\sum_{k=0}^{u+1} g_{ij}(k)\phi_j(u+1-k) + q_1\sum_{j=1}^{2}\sum_{k=0}^{u} g_{ij}(k)\phi_j(u-k), \quad a_1 \le u < a_2, \tag{3.2}$$

$$\begin{aligned}\phi_i(u) = p_1p_2\sum_{j=1}^{2}\sum_{k=0}^{u+1} g_{ij}(k)\phi_j(u+1-k) + (p_1q_2 + q_1p_2)\sum_{j=1}^{2}\sum_{k=0}^{u} g_{ij}(k)\phi_j(u-k)\\ + q_1q_2\sum_{j=1}^{2}\sum_{k=0}^{u-1} g_{ij}(k)\phi_j(u-1-k), \quad u \ge a_2.\end{aligned} \tag{3.3}$$

Let $\widetilde{\phi}_i(s)$ and $\widetilde{g}_{ij}(s)$ be the probability generating function of $\phi_i(k)$ and $g_{ij}(k)$. We multiply both sides of the equation (3.1)(3.2)(3.3) by s^{u+1}, and sum u from 0 to ∞. We have

$$s\widetilde{\phi}_i(s) = (p_1+q_1s)(p_2+q_2s)\sum_{j=1}^{2}\widetilde{g}_{ij}(s)\widetilde{\phi}_j(s) - p_1p_2\sum_{j=1}^{2} g_{ij}(0)\phi_j(0) + \sum_{u=0}^{a_1-1} E_i(s)s^{u+1} + \sum_{u=0}^{a_2-1} N_i(s)s^{u+1},$$

for $i = 1, 2$,

$$E_i(u) = q_1\sum_{j=1}^{2}\sum_{k=0}^{u+1} g_{ij}(k)\phi_j(u+1-k) - q_1\sum_{j=1}^{2}\sum_{k=0}^{u} g_{ij}(k)\phi_j(u-k), \quad u = 0, 1, 2, \cdots, a_1 - 1,$$

$$\begin{aligned}N_i(u) = p_1q_2\sum_{j=1}^{2}\sum_{k=0}^{u+1} g_{ij}(k)\phi_j(u+1-k) + q_2(q_1 - p_1)\sum_{j=1}^{2}\sum_{k=0}^{u} g_{ij}(k)\phi_j(u-k)\\ + q_1q_2\sum_{j=1}^{2}\sum_{k=0}^{u-1} g_{ij}(k)\phi_j(u-1-k), \quad u = 0, 1, 2, \cdots, a_2 - 1,\end{aligned}$$

let

$$A(p_i, s) = p_i + (1 - p_i)s = p_i + q_is, \quad e_i = \sum_{j=1}^{2} g_{ij}(0)m_j(0),$$

$$F_i(s) = \sum_{u=0}^{a_1-1} E_i(u)s^{u+1}, \quad M_i(s) = \sum_{u=0}^{a_2-1} N_i(u)s^{u+1}, \quad H_i(s) = p_1p_2e_i - M_i(s) - F_i(s).$$

Then we have

$$\begin{cases}[A(p_1, s)A(p_2, s)\widetilde{g}_{11}(s) - s]\widetilde{\phi}_1(s) + A(p_1, s)A(p_2, s)\widetilde{g}_{12}(s)\widetilde{\phi}_2(s) = H_1(s),\\ A(p_1, s)A(p_2, s)\widetilde{g}_{21}(s)\widetilde{\phi}_1(s) + [A(p_1, s)A(p_2, s)\widetilde{g}_{22}(s) - s]\widetilde{\phi}_2(s) = H_2(s).\end{cases} \tag{3.4}$$

From (3.4) we can know

$$\{[A(p_1,s)A(p_2,s)\widetilde{g}_{11}(s)-s][A(p_1,s)A(p_2,s)\widetilde{g}_{22}(s)-s]-A^2(p_1,s)A^2(p_2,s)\widetilde{g}_{12}(s)\widetilde{g}_{21}(s)\}\widetilde{\phi}_1(s)$$
$$=H_1(s)[A(p_1,s)A(p_2,s)\widetilde{g}_{22}(s)-s]-H_2(s)A(p_1,s)A(p_2,s)\widetilde{g}_{12}(s). \tag{3.5}$$

For notational convenience, we define

$$h_i(0)=p_1p_2e_i,\ \overline{g}_{ij}(0)=p_1p_2g_{ij}(0),\quad \overline{g}_{ij}(1)=p_1p_2g_{ij}(1)+(q_1p_2+p_1q_2)g_{ij}(0),\quad i\neq j,$$
$$\overline{g}_{ii}(1)=p_1p_2g_{ii}(1)+(q_1p_2+p_1q_2)g_{ii}(0)-1,$$
$$\overline{g}_{ij}(k)=p_1p_2g_{ij}(k)+(q_1p_2+p_1q_2)g_{ij}(k-1)+q_1q_2g_{ij}(k-2),\ k\in\mathbb{N}\setminus\{0,1\},$$
$$h_i(k)=-N_i(k-1)-E_i(k-1),\quad k=1,2,\cdots,a_1,\quad h_i(k)=-N_i(k-1),$$
$$k=a_1+1,a_1+2,\cdots,a_2,\quad h_i(k)=0,\quad k=a_2+1,a_2+2,\cdots,i=1,2,$$
$$f_k=\sum_{n=0}^{k}[\overline{g}_{11}(n)\overline{g}_{22}(k-n)-\overline{g}_{21}(n)\overline{g}_{12}(k-n)],$$
$$g_k^{(1)}=\sum_{n=0}^{k}\phi_1(n)f_{k-n},\quad A_k^{(1)}=\sum_{n=0}^{k}[h_1(n)\overline{g}_{22}(k-n)-h_2(n)\overline{g}_{12}(k-n)],\quad k\in\mathbb{N}.$$

Let $\widetilde{g}^{(1)}(s)$, $\widetilde{f}(s)$, and $\widetilde{A}^{(1)}(s)$ be the generating functions of $g_k^{(1)}$, f_k, and $A_k^{(1)}$ respectively. According to the property of generating function, we can obtain from (3.5)

$$\widetilde{g}^{(1)}(s)=\widetilde{f}(s)\widetilde{\phi}_1(s)=\widetilde{A}^{(1)}(s). \tag{3.6}$$

The above equality can be obtained as

$$\sum_{n=0}^{k}\phi_1(n)f_{k-n}=A_k^{(1)},\quad k\in\mathbb{N}. \tag{3.7}$$

Similarly, we can get

$$\sum_{n=0}^{k}\phi_2(n)f_{k-n}=A_k^{(2)},\quad k\in\mathbb{N}. \tag{3.8}$$

Theorem 1. For $i=1,2$ and $k\in\mathbb{N}^+$, the ruin probability satisfies the recursive formula as follows

$$\phi_i(k)=\begin{cases}\frac{1}{f_0}[A_k^{(i)}-\sum_{n=0}^{k-1}\phi_i(n)f_{k-n}], & \text{if } f_0\neq 0,\\ \frac{1}{f_1}[A_{k+1}^{(i)}-\sum_{n=0}^{k-1}\phi_i(n)f_{k+1-n}], & \text{if } f_0=0,\ f_1\neq 0.\end{cases} \tag{3.9}$$

Proof. From (3.7) and (3.8), we just need to show that the positive security loading condition does not hold when $f_0=f_1=0$. Note that

$$f_1=p_1p_2g_{11}(0)[p_1p_2g_{22}(1)+q_1p_2g_{22}(0)+p_1q_2g_{22}(0)-1]-p_1p_2g_{21}(0)[p_1p_2g_{12}(1)$$
$$+q_1p_2g_{12}(0)+p_1q_2g_{12}(0)]+p_1p_2g_{22}(0)[p_1p_2g_{11}(1)+q_1p_2g_{11}(0)+p_1q_2g_{11}(0)-1]$$
$$-p_1p_2g_{12}(0)[p_1p_2g_{21}(1)+q_1p_2g_{21}(0)+p_1q_2g_{21}(0)]\leq 0.$$

If $f_1=0$, then $g_{11}(0)=g_{22}(0)=0$ and

$$p_1p_2g_{21}(0)[p_1p_2g_{12}(1)+q_1p_2g_{12}(0)+p_1q_2g_{12}(0)]=0, \tag{3.10}$$

$$p_1p_2g_{12}(0)[p_1p_2g_{21}(1)+q_1p_2g_{21}(0)+p_1q_2g_{21}(0)]=0. \tag{3.11}$$

From $f_0 = p_1^2 p_2^2 (g_{11}(0)g_{22}(0) - g_{21}(0)g_{12}(0)) = 0$, we know $g_{21}(0)g_{12}(0) = 0$.

Consequently, we need to take two situations into account.

(i) $g_{12}(0) = 0$ but $g_{21}(0) \neq 0$.
(ii) $g_{21}(0) = 0$ but $g_{12}(0) \neq 0$.

Since the remaining steps of the proof are the same as those in Chen et al.[4], we omit the details here. Finally, from (3.7)(3.8), we can obtain the recursive formula. Theorem 1 has been proved. □

For the sake of obtaining the ruin probability, we have to get the survival probability by theorem 1. In consideration of the premises, we need the initial $\phi_i(0)$, $i = 1, 2$. Next we have to find two equations between them. In this section, we only consider the case with $a_2 = 0$.
Define

$$d_i(0) = \phi_i(0), \quad d_i(u) = \phi_i(u) - \phi_i(u-1), \quad u \geq 1,$$
$$B_i(0) = A_0^{(i)}, \quad B_i(u) = A_u^{(i)} - A_{u-1}^{(i)}, \quad u \geq 1.$$

As showed in [4], we have

$$\widetilde{f}(s)\widetilde{d}_i(s) = \widetilde{B}_i(s), \quad i = 1, 2. \tag{3.12}$$

Therefore

$$\widetilde{B}_i'(s) = \widetilde{f}'(s)\widetilde{d}_i(s) + \widetilde{f}(s)\widetilde{d}_i'(s), \quad i = 1, 2. \tag{3.13}$$

On account of

$$\widetilde{d}_i(s) = \sum_{u=0}^{\infty} s^u d_i(u), \quad \widetilde{d}_i(1) = \sum_{u=0}^{\infty} d_i(u) = \lim_{n\to\infty} \phi_i(n) = 1, \quad \widetilde{B}_i(1) = \lim_{n\to\infty} A_n^{(i)} = 0,$$

from(3.12)(3.13), we know $\widetilde{f}(1) = 0$ and $\widetilde{B}_1'(s) = \widetilde{f}_1'(s)$. $A_k^{(1)}$ can be rewritten

$$A_k^{(1)} = \xi_k^{(1)}\phi_1(0) + \eta_k^{(1)}\phi_2(0), \quad k \in \mathbb{N}, \tag{3.14}$$

where

$$\xi_k^{(1)} = p_1p_2(g_{11}(0)\overline{g}_{22}(k) - g_{21}(0)\overline{g}_{12}(k)), \quad \eta_k^{(1)} = p_1p_2(g_{12}(0)\overline{g}_{22}(k) - g_{22}(0)\overline{g}_{12}(k)).$$

So we have

$$\begin{aligned}
\widetilde{B}_1'(1) &= \sum_{k=0}^{\infty} kB_1(k) = -\phi_1(0)\sum_{i=0}^{\infty}\xi_i^{(1)} - \phi_2(0)\sum_{i=0}^{\infty}\eta_i^{(1)} \\
&= p_1p_2\phi_1(0)[g_{11}(0)p_{21} + g_{21}(0)p_{12}] + p_1p_2\phi_2(0)[g_{12}(0)p_{21} + g_{22}(0)p_{12}].
\end{aligned}$$

On the other hand

$$\begin{aligned}
\widetilde{f}'(1) &= \sum_{k=0}^{\infty} kf_k = \sum_{k=0}^{\infty}\sum_{n=0}^{k} k[\overline{g}_{11}(n)\overline{g}_{22}(k-n) - \overline{g}_{21}(n)\overline{g}_{12}(k-n)] \\
&= \sum_{n=0}^{\infty}\sum_{k=n}^{\infty} k[\overline{g}_{11}(n)\overline{g}_{22}(k-n) - \overline{g}_{21}(n)\overline{g}_{12}(k-n)] \\
&= \sum_{n=0}^{\infty}\overline{g}_{11}(n)\sum_{k=0}^{\infty}k\overline{g}_{22}(k) + \sum_{n=0}^{\infty}n\overline{g}_{11}(n)\sum_{k=0}^{\infty}\overline{g}_{22}(k) \\
&\quad - \sum_{n=0}^{\infty}\overline{g}_{21}(n)\sum_{k=0}^{\infty}k\overline{g}_{12}(k) - \sum_{n=0}^{\infty}n\overline{g}_{21}(n)\sum_{k=0}^{\infty}\overline{g}_{12}(k) \\
&= p_{12}(1 - q_1 - q_2 - \mu_2) + p_{21}(1 - q_1 - q_2 - \mu_1).
\end{aligned}$$

So we obtain

$$\begin{aligned}&p_1p_2\phi_1(0)[g_{11}(0)p_{21}+g_{21}(0)p_{12}]+p_1p_2\phi_2(0)[g_{12}(0)p_{21}+g_{22}(0)p_{12}]\\&=p_{12}(1-q_1-q_2-\mu_2)+p_{21}(1-q_1-q_2-\mu_1).\end{aligned}\tag{3.15}$$

Hence we find a relationship between $\phi_1(0)$ and $\phi_2(0)$. In the next moment, we are going to find another relationship between $\phi_1(0)$ and $\phi_2(0)$. To do it, we consider several cases of $f_0 = p_1^2p_2^2(g_{11}(0)$ $g_{22}(0) - g_{21}(0)g_{12}(0))$. To begin with, we consider $f_0 = 0$. $\phi_1(0)f_1 = A_1^{(1)}$ can be obtained from (3.7), and

$$K_1\phi_1(0)+K_2\phi_2(0)=0,\tag{3.16}$$

where

$$\begin{aligned}K_1&=p_1^2p_2^2[g_{11}(1)g_{22}(0)-g_{12}(0)g_{21}(1)]-p_1p_2g_{22}(0)\le 0,\\K_2&=p_1^2p_2^2[g_{22}(0)g_{12}(1)-g_{12}(0)g_{22}(1)]+p_1p_2g_{12}(0)\ge 0.\end{aligned}$$

Since $K_1 = K_2 = 0$ holds if and only if $g_{12}(0) = g_{22}(0) = 0$. At this time, $e_1 = g_{11}(0)\phi_1(0)$, $e_2 = g_{21}(0)\phi_1(0)$. Therefore

$$f_1\phi_2(0)=[p_1^2p_2^2(g_{11}(1)g_{21}(0)-g_{11}(0)g_{21}(1))-p_1p_2g_{21}(0)]\phi_1(0).\tag{3.17}$$

Secondly, we consider $f_0 > 0$.

$$\begin{aligned}\widetilde{f}'(s)=&[(q_1A(p_2,s)+q_2A(p_1,s))\widetilde{g}_{11}(s)+A(p_1,s)A(p_2,s)\widetilde{g}_{11}'(s)-1][A(p_1,s)A(p_2,s)\widetilde{g}_{22}(s)-s]\\&+[(q_1A(p_2,s)+q_2A(p_1,s))\widetilde{g}_{22}(s)+A(p_1,s)A(p_2,s)\widetilde{g}_{22}'(s)-1][A(p_1,s)A(p_2,s)\widetilde{g}_{11}(s)-s]\\&-2A(p_1,s)A(p_2,s)(q_1A(p_2,s)+q_2A(p_1,s))\widetilde{g}_{12}(s)\widetilde{g}_{21}(s)\\&-A^2(p_1,s)A^2(p_2,s)[\widetilde{g}_{12}'(s)\widetilde{g}_{21}(s)+\widetilde{g}_{21}'(s)\widetilde{g}_{12}(s)].\end{aligned}$$

As aforementioned,

$$\begin{aligned}\widetilde{f}'(1)&=-p_{21}[(2-p_1-p_2)p_{11}+\mu_{11}-1]-p_{12}[(2-p_1-p_2)p_{22}+\mu_{22}-1]\\&\quad-2(2-p_1-p_2)p_{12}p_{21}-\mu_{12}p_{21}-\mu_{21}p_{12}\\&=-(2-p_1-p_2)(p_{21}+p_{12})-(\mu_1p_{21}+\mu_2p_{12})+p_{21}+p_{12}\\&=(p_1+p_2-1)(p_{21}+p_{12})-(\mu_1p_{21}+\mu_2p_{12})>0.\end{aligned}$$

Consequently, there is a $\rho \in (0,1)$ because of $\widetilde{f}(1) = 0$, which makes $\widetilde{f}(\rho) = 0$, i.e., $\widetilde{A}^{(1)}(\rho) = 0$. That is

$$\begin{aligned}&\{[g_{11}(0)\widetilde{g}_{22}(\rho)-g_{21}(0)\widetilde{g}_{12}(\rho)]A(p_1,\rho)A(p_2,\rho)-g_{11}(0)\rho\}\phi_1(0)\\&+\{[g_{12}(0)\widetilde{g}_{22}(\rho)-g_{22}(0)\widetilde{g}_{12}(\rho)]A(p_1,\rho)A(p_2,\rho)-g_{12}(0)\rho\}\phi_2(0)=0.\end{aligned}\tag{3.18}$$

Thirdly, $f_0 < 0$ should be considered. Furthermore $\widetilde{f}(0) = f_0 < 0$. And

$$\begin{aligned}\widetilde{f}(-1)&=[(2p_1-1)(2p_2-1)\widetilde{g}_{11}(-1)+1][(2p_1-1)(2p_2-1)\widetilde{g}_{22}(-1)+1]\\&\quad-(2p_1-1)^2(2p_2-1)^2\widetilde{g}_{21}(-1)\widetilde{g}_{12}(-1)>(1-\widetilde{g}_{11}(1))(1-\widetilde{g}_{22}(1))-\widetilde{g}_{21}(1)\widetilde{g}_{12}(1)\\&=(1-p_{11})(1-p_{22})-p_{21}p_{12}=0,\quad\forall p_i\in(0,1],i=1,2.\end{aligned}$$

In this way, there is a $\rho \in (-1,0)$ that makes $\widetilde{f}(\rho) = 0$, i.e., $\widetilde{A}^{(1)}(\rho) = 0$. This means that (3.18) still holds.

4 Numerical Illustration

In the following example, we investigate the effect on the dividend payments on ruin probability. We let $a_2 = 0$, $p_1 = 0.95$, $p_2 = 0.85, 0.87, 0.89, 0.91$. The numerical analysis of this section only consider $f_0 = 0$. Other situations are similar. We start with the example considered by Janssen and Reinhard [1][2]. The distribution of claims $g_{ij}(k)$ is given in Table 1. We aim to compute the ruin probabilities with different values of the p_1 and p_2. From the previous statement, we can see that if we obtain the initial values, we will get the ruin probabilities by (3.9). The results are given in Table 2.

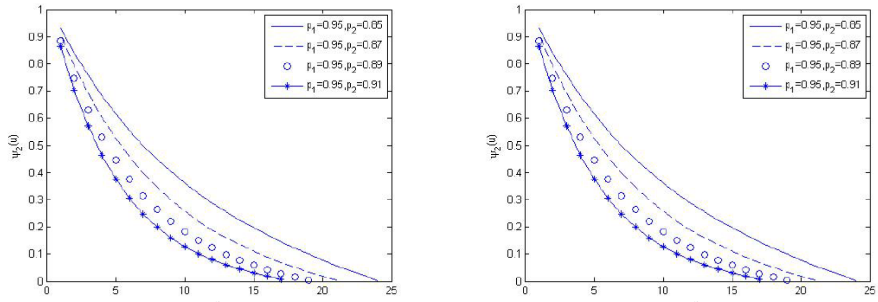
Table 1. The distribution of claims

k	$g_{11}(k)$	$g_{12}(k)$	$g_{21}(k)$	$g_{22}(k)$
0	5/8	0	0	0
1	1/8	1/8	0	1/6
2	1/8	0	1/2	1/6
3	0	0	1/6	0
≥ 4	0	0	0	0

The table 2 provides several important points of $\psi_i(u)$,$i = 1, 2$ in regard to the initial surplus u for fixed p_1, p_2. As is revealed in the table 2, we can see clearly that $\psi_i(u)$ increases as p_1, p_2 decrease while $\psi_i(u)$ decreases as p_1, p_2 increase. From another aspect, $\psi_i(u)$ always decreases as the value of u increase. Furthermore, in order to more vividly reflect the influence of the parameters on ruin probability, we provide Figs 1a and 1b, which show the impact of p_i,q_i and the initial surplus u on ruin probability, respectively. The figures lead us to the conclusion that random dividends to shareholders and policyholders play an important role in the ruin probability.

Table 2. The values of $\psi_i(u)(p_1 = 0.95)$.

	$\psi_1(u)$				$\psi_2(u)$			
u	$p_2 = 0.85$	$p_2 = 0.87$	$p_2 = 0.89$	$p_2 = 0.91$	$p_2 = 0.85$	$p_2 = 0.87$	$p_2 = 0.89$	$p_2 = 0.91$
0	0.8514	0.8088	0.7682	0.7293	1	1	1	1
1	0.7693	0.7068	0.6485	0.5942	0.9307	0.9084	0.8859	0.8633
2	0.6937	0.6160	0.5460	0.4827	0.8401	0.7928	0.7469	0.7023
3	0.6256	0.5370	0.4597	0.3921	0.7575	0.6910	0.6288	0.5704
4	0.5639	0.4679	0.3868	0.3183	0.6831	0.6024	0.5294	0.4635
5	0.5079	0.4072	0.3250	0.2580	0.6159	0.5250	0.4456	0.3764
6	0.4570	0.3529	0.2725	0.2086	0.5551	0.4572	0.3748	0.3054
7	0.4106	0.3068	0.2278	0.1679	0.4998	0.3978	0.3148	0.2473
8	0.3681	0.2652	0.1896	0.1344	0.4496	0.3455	0.2637	0.1998
9	0.3291	0.2282	0.1568	0.1066	0.4037	0.2994	0.2202	0.1606
10	0.2933	0.1953	0.1284	0.0834	0.3618	0.2585	0.1830	0.1283
11	0.2602	0.1657	0.1039	0.0639	0.3233	0.2222	0.1511	0.1014
12	0.2295	0.1392	0.0824	0.0474	0.2878	0.1898	0.1234	0.0790
13	0.2009	0.1152	0.0635	0.0334	0.2550	0.1608	0.0994	0.0601
14	0.1742	0.0933	0.0469	0.0212	0.2247	0.1347	0.0784	0.0442
15	0.1492	0.0734	0.0320	0.0107	0.1964	0.1110	0.0600	0.0305
16	0.1256	0.0550	0.0186	0.0014	0.1699	0.0895	0.0436	0.0187
17	0.1033	0.0380	0.0064	0	0.1451	0.0698	0.0290	0.0084
18	0.0821	0.0222	0	0	0.1217	0.0516	0.0159	0
19	0.0618	0.0073	0	0	0.0995	0.0348	0.0039	0
20	0.0424	0	0	0	0.0784	0.0191	0	0
21	0.0238	0	0	0	0.0583	0.0045	0	0
22	0.0058	0	0	0	0.0390	0	0	0
23	0	0	0	0	0.0995	0	0	0
24	0	0	0	0	0.0025	0	0	0



(a) The ruin probability of $\psi_1(u)$. (b) The ruin probability of $\psi_2(u)$.

Fig. 1. The ruin probability of $\psi_i(u)$

5 Conclusions

In this paper, we mainly investigate the discrete semi-Markov risk model with random dividends paying to shareholders and policyholders. The main feature of this paper is that we derive recursive formulae for the ruin probabilities with different values of the p_1 and p_2 by using the method of probability generating function. Numerical example is also presented to illustrate the effect of the related parameters on the ruin probabilities.

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Competing Interests

Author has declared that no competing interests exist.

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