



# Rate Decline-based Models for Gas Reservoir Performance Prediction in Niger Delta Region

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## Authors' contributions

This work was carried out in collaboration between all authors. Author ANO designed the study, analyzed the results and intellectual content in the manuscript. Author DTO designed the study, managed the literature searches and wrote the first draft of the manuscript. Author JUA reviewed the analyses of the results and important intellectual content in the manuscript. All authors read and approved the final manuscript.

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## ABSTRACT

This work considers the Decline Curve Analysis (DCA) approach as a quick tool to estimate the gas reservoir performance of field "ABC" in the Niger Delta region. The conventional Arps' models: Exponential, Harmonic and Hyperbolic, alongside with the Reciprocal and Quadratic models were used. Production data: gas production rate ( $q_t$ ) and gas cumulative production ( $G_p$ ) were obtained from 13 wells in the field "ABC". Multivariate analyses were performed with the mentioned models to establish the decline constant ( $D_i$ ) and decline exponent ( $b$ ); for hyperbolic model, of the field "ABC" in the Niger Delta region. A decline constant of  $0.000064\text{day}^{-1}$  was obtained from all the models with exception of Reciprocal model with  $0.00053\text{day}^{-1}$  for the gas field. Also, the decline exponent ( $b$ ) obtained for Hyperbolic model was 0.9999. The statistical analysis: absolute error, standard deviation and coefficient of determination, of the fitted models used to ascertain the extent of their predicted values differ from the field test data results in Arps' models: Exponential - 0.1150,

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0.02666 and 0.9981; Harmonic - 0.11547, 0.02665 and 0.9982 and Hyperbolic - 0.11547, 0.02665 and 0.9982, respectively. Furthermore, Reciprocal and Quadratic models generated an absolute error, standard deviation and coefficient of determination of 0.09726, 0.026745 and 0.9911, and 0.0097, 0.000008 and 0.9998, respectively. Thus, the results indicate that, modern rate decline models for reservoir performance analysis can compete with the well-known Arps' model(s). Therefore, the fitted Quadratic-based model can be used as a quick tool to analyze the reservoir performance of the gas field "ABC" in the Niger Delta region.

*Keywords: Rate decline analysis; gas reservoir; Arps' models; Reciprocal model; Quadratic model; Niger Delta region.*

## 1. INTRODUCTION

In petroleum engineering, Decline Curve Analysis (DCA) is the most used reserves estimation approach when historic production data are available and sufficient to establish a trend. The most popular decline trend is that which represents the decline in the hydrocarbon production rates with time; another is production rate against cumulative hydrocarbon production [1]. Arps in 1945 created the foundation of decline curve analysis by proposing simple mathematical curves: exponential, harmonic and hyperbolic, as a tool for creating a reasonable outlook for the production of an oil well once it has reached the onset of decline [2]. Rate decline analysis is established on the postulation that the production history of hydrocarbon reservoir(s) and factors causing the historical decline continue unchanged during the forecast period. These factors include both reservoir conditions and operating conditions. Some of the reservoir factors that affect the decline rate are pressure depletion, number of producing wells, drive mechanism, reservoir characteristics, saturation changes and relative permeability [3]. Additionally, the operating conditions that influence the decline rate include: separator pressure, tubing size, choke setting, workovers, compression, operating hours and artificial lift [4]. Although the decline rate comes with its drawbacks, the biggest advantage of decline curve analysis is that it is virtually independent of the size and shape of the reservoir or the actual drive mechanism [5]. In other words, the detailed description of the reservoir or production data is not required to perform the rate decline analysis. Decline curves of various forms can be used to create significant outlooks for fluid production of a single well or an entire field. Conversely, it should be emphasized that in many field cases a single curve is not sufficient to obtain a good fit and it may be necessary to use a combination of curves to obtain good agreement [6]. A great number of models for decline rate analysis are

heuristic and still based on Equation of Arps [7]; presented in equation 1 [8], for oil wells during pseudo steady-state period. Fetkovitch [9] presented another approach - type curve, to analyze production data. This type curve consist of two portions: transient and boundary dominated production periods. The transient portion comes from constant pressure type curve developed by van Everdingen [10] while the boundary dominated portion is the same as Arps [7] depletion stems. Arps [7] and Fetkovitch [9] models are derived empirically; however, Arps' models are still the preferred method for forecasting oil production and proven reserves [11]. Fetkovitch's method calculates the ultimate recovery, but it is constrained to existing operating conditions earlier alluded. Further works by Blasingame and Lee [12] and Agarwal et al. [13] are similar to Fetkovitch's type-curves for analysis of production data. The major difference is the incorporation of flowing pressure data along with production rate to solve for hydrocarbon in-place analytically. On the other hand, Arps' approach and its modified varieties have been used widely to estimate future performance of hydrocarbon reserves all over the world. It remained so until of recent when new methods such as Reciprocal method [14] and Quadratic method [15] were introduced into the computation of rate decline analysis. The Reciprocal rate method presumes that flowing well bottom-hole pressure is approximately constant and was used to estimate hydrocarbon reserves using only rate-time production data. This model requires a plot of the reciprocal of flow rate ( $q^{-1}$ ) against the cumulative production to flowrate ratio ( $G_p/q$ ) as presented by Equation 2. Also, Johnson et al. [15] developed a method which makes use of the Semi-analytical formulation by Blasingame and Rushing [16] and Empirical formulation by Ilk et al. [17]. He modified the equations to yield the relation expanded in Equation 3. Though, the aforementioned methods have been tested and

validated to be effective in hydrocarbon reserves estimation in several regions of the world, they are yet to be used as much as the Arps' approach especially in the Niger Delta region. This is due to the fact that developed models available in literature for gas reservoir performance analyses were not based on Niger Delta data. Additionally, their prediction results may not be that accurate, as the mentioned limitation of the available models poses a major challenge on the prediction of the gas reservoir performance using models in the literature. Therefore in this paper, rate decline-based models were fitted and validated for use as quick tools for gas reservoirs performance predictions in the Niger Delta region.

$$q_t = \frac{q_i}{(1+b D_i t)^{1/b}} \quad (1)$$

where:

$q_t$  = gas flow rate at time t, MMscf/day

$q_i$  = initial gas flow rate, MMscf/day

t = time, days

$D_i$  = decline constant, day<sup>-1</sup>

b = decline exponent

$$\frac{1}{q_t} = \frac{1}{q_i} + \frac{D_i}{q_i} \left( \frac{G_p}{q_t} \right) \quad (2)$$

$$q_t = q_i - D_i G_p + \frac{1}{2} \frac{D_i}{G} G_p^2 \quad (3)$$

Additionally, analysis of Equation 3 indicates that a plot of  $\frac{q_i - q_t}{G_p}$  against  $G_p$  yield a straight line with an intercept equal to the decline constant ( $D_i$ ) and slope as  $\frac{D_i}{2G}$ . Also, the extrapolation of  $\frac{q_i - q_t}{G_p} = 0$ , gives  $G_{pmax} = 2G$ .

Where  $G$  approximate equivalent to  $G_{pmax} = \frac{q_i}{D_i}$ . Thus,  $G$  is expanded as;

$$G = \frac{q_i}{2D_i} \quad (4)$$

where:

$G_p$  = Cumulative gas production, MMscf

$G_{pmax}$  = Maximum Cumulative gas that can be produced, MMscf

G = Original Gas in place, MMscf

## 2. MATERIALS AND METHODS

### 2.1 Data Acquisition and Models Fitting

The production data: gas production rate ( $q_t$ ) and gas cumulative production ( $G_p$ ) of the gas field "ABC" in the Niger Delta region was obtained from 13 wells. The range of these gas production data is presented in Table 1. Multivariate analyses were performed based on the existing rate decline models: Arps (i.e., Exponential, Harmonic and Hyperbolic), Reciprocal and Quadratic model to determine the decline constant ( $D_i$ ) and exponent (b) - in terms of Hyperbolic model for the "ABC" gas field. The general reduced gradient (GRG) iteration protocol in the Microsoft Excel Solver was used to fit the aforementioned rate decline models. The fitted models are presented in Table 2.

**Table 1. Summary of production data**

Type of data	Range
Flow rate ( $q_t$ ), MMscf/day	113.79 – 600.52
Cumulative production ( $G_p$ ), MMscf	113.79 – 379080.20
Number of wells producing	13

**Table 2. Rate decline fitted models**

S/N	Model	Flow Rate ( $q_t$ ); MMscf	Cumulative production ( $G_p$ ); MMscf
1.	Reciprocal	$\frac{1}{q_t} = \frac{1}{q_i} + \frac{0.00053}{q_i} \left( \frac{G_p}{q_t} \right)$	$G_{p(t)} = 1886.7(q_i - q_t)$
2.	Exponential	$q_t = q_i \exp(-0.000064t)$	$G_{p(t)} = 15625(q_i - q_t)$
3.	Harmonic	$q_t = \frac{q_i}{(1 + 0.000064t)}$	$G_{p(t)} = 15625 q_i \ln \left( \frac{q_i}{q_t} \right)$
4.	Quadratic	$q_t = q_i - 6.4 \times 10^{-5} G_p + 3.4 \times 10^{-13} G_p^2$	$G_{p(t)} = 15625(q_i - q_t)$
5.	Hyperbolic	$q_t = \frac{q_i}{(1 + 0.000064t)^{\frac{1}{0.9999}}}$	$G_{p(t)} = 1.6 \times 10^{10} q_i \left( 1 - \left( \frac{q_t^{0.00001}}{q_i^{0.00001}} \right) \right)$

**Table 3. Statistical validation analysis**

S/N	Validation tools	Reciprocal	Quadratic	Exponential	Harmonic	Hyperbolic
1	Average error ( $E_{avg}$ )	0.01772	0.0097	-0.1150	-0.11547	-0.11547
2	Absolute error ( $E_{abs}$ )	0.09726	0.0097	0.1150	0.11547	0.11547
3	Root mean square error ( $E_{rms}$ )	0.01176	0.00013	0.0162	0.2354	0.01627
4	Normalized root mean square error ( $E_{nrms}$ )	0.00000003	0.0000	0.00000004	0.0000006	0.00000004
5	Coefficient of determination ( $r^2$ )	0.9911	0.9998	0.9981	0.9982	0.9982
6	Standard Deviation (SD)	0.026745	0.000008	0.02666	0.02665	0.02665
7	Normalized Standard Deviation (NSD)	10.85	0.01143	12.736	12.698	12.698

## 2.2 Models Comparison and Validation

The various fitted models' predictions were compared with the obtained field test data from the gas field "ABC". The parameters considered for comparison were field test production rate ( $q_{t_{field}}$ ) and fitted model predicted production rate ( $q_{t_{model}}$ ) against time ( $t$ ), field test cumulative production ( $G_{P_{field}}$ ) versus fitted model predicted cumulative production ( $G_{P_{model}}$ ), and field test cumulative production ( $G_{P_{field}}$ ) and fitted model predicted cumulative production ( $G_{P_{model}}$ ) against time ( $t$ ). In addition to these comparisons, statistical analyses were performed to validate the reliability of the fitted rate decline models' forecasted or predicted values. The statistical methods used are the average error ( $E_{avg}$ ), absolute error ( $E_{abs}$ ), root mean square error ( $E_{rms}$ ), normalized root mean square error ( $E_{nrms}$ ), coefficient of determination ( $r^2$ ), standard deviation (SD) and normalized standard deviation (NSD). Thus, their respective mathematical equations are expanded in Appendix A. The results of the statistical analyses are presented in Table 3 above.

## 3. RESULTS AND DISCUSSION

As earlier alluded, multivariate analyses were performed with the obtained field data to determine the decline constant of the "ABC" gas field in the Niger Delta region. The established decline constant ( $D_i$ ) for the "ABC" gas field is  $0.000064\text{day}^{-1}$  for all the models except for Reciprocal model which establish decline constant ( $D_i$ ) of  $0.00053\text{day}^{-1}$ . In addition, the hyperbolic approach - Arps' model establish a decline exponent ( $b$ ) of 0.9999 for the "ABC" gas field. This decline exponent for the Arps' models

implies that, harmonic and hyperbolic models will predict similar production values (data) for the "ABC" gas field; as the decline exponent for harmonic model is unity (1). The similarity of the two models predictions are observed in their statistical analyses presented in Table 3.

### 3.1 Arps' Models

Figs. 1 through 6 present the results obtained based on the Arps' models. Figs. 1 and 2 depict the exponential approach for the rate decline analysis. The comparison of the fitted model's prediction (i.e., production rate – time) with field data in Fig. 1 indicates an alignment of the predicted production rate – time with the actual field data. Also, Fig. 2 presents the comparison of the field cumulative production ( $G_{P_{field}}$ ) with the predicted cumulative production ( $G_{P_{model}}$ ) based on Arps' exponential model. The comparison of these results indicates close predictions of the "ABC" field data. The degree of this close prediction is indicated in the coefficient of determination ( $r^2$ ) of 0.9981. Furthermore, Fig. B-1 in Appendix B depicts the cumulative production – time comparison for the field data and fitted model prediction. The result indicates close alignment of the predicated data with the actual field data. On the other hand, the harmonic and hyperbolic models with about the same decline exponent ( $b$ ) of unity have the same predictions. Figs. 3 and 5 indicate the alignment of the predicted production rate with the field production rate for harmonic and hyperbolic models, respectively. Similarly, Figs. 4 and 6 present the comparison the field cumulative production and predicted cumulative production for harmonic and hyperbolic model, respectively. These models' predictions resulted in coefficient of determination of 0.9982 with the

field data; an indication of close prediction of the actual field data. Additionally, Figs. B-2 and B-3 presents similar results for the cumulative production – time comparison for the field data and fitted model prediction.

### 3.2 Reciprocal Model

Figs. 7 and 8 depicts the obtained results for the comparison of field production rate ( $q_{t_{field}}$ ) and predicated production rate ( $q_{t_{model}}$ ) – time, and field cumulative production ( $G_{P_{field}}$ ) against predicted cumulative production ( $G_{P_{model}}$ ), respectively. The predicted production rate from the fitted Reciprocal model resulted in steady decline; as observed in Fig. 7. This prediction tends to align with the field data at the early year

of production, but later shows disparity. This is attributed to the reciprocal nature of the model's production rate. However, the predicted cumulative gas production ( $G_{P_{model}}$ ) compared with the actual field cumulative gas production ( $G_{P_{field}}$ ) with coefficient of determination ( $r^2$ ) of 0.9911 as shown in Fig. 8. Conversely, it is worth noting that the fitted reciprocal model predictions (cumulative production) for the “ABC” gas field are accurate in the early years of production; as observed in Fig. B-4. This observation is due to the reciprocal or inverse of the production rate in the model. This approach restrict the flexibility of the model; especially in the production rate prediction, since the reciprocated production rate value(s) is/are return to normal form to compare with the actual field data.

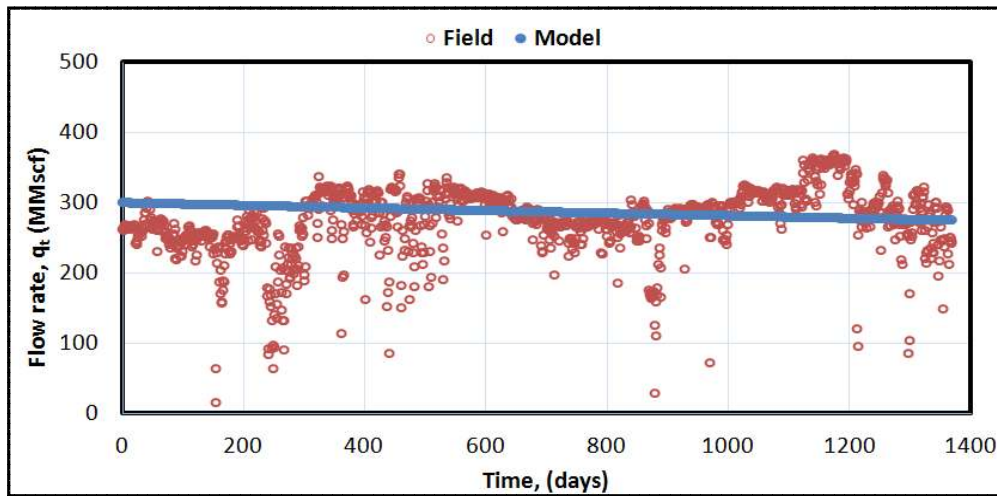


Fig. 1. Gas production rate - Time plot (Exponential model)

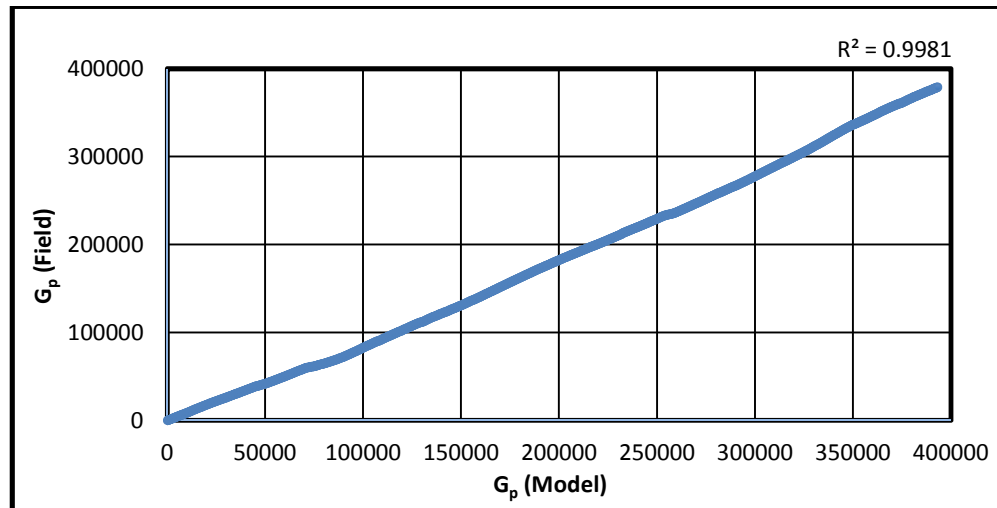


Fig. 2. Field Data – Model predicted cumulative production plot (Exponential model)

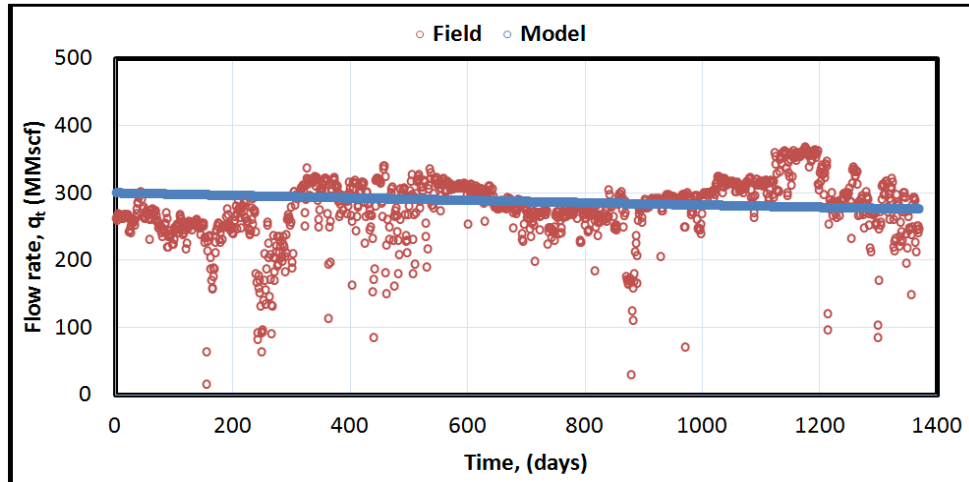


Fig. 3. Gas production rate - Time plot (Harmonic model)

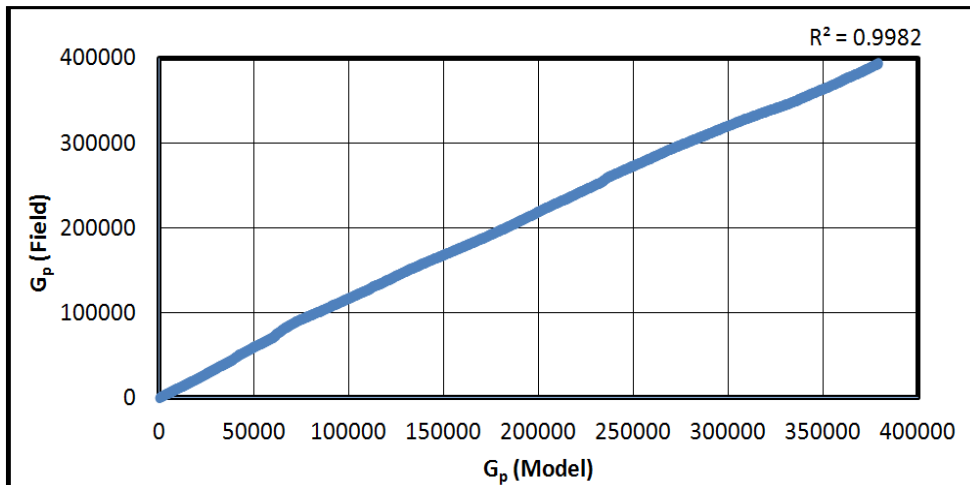


Fig. 4. Field data – Model predicted cumulative production plot (Harmonic model)

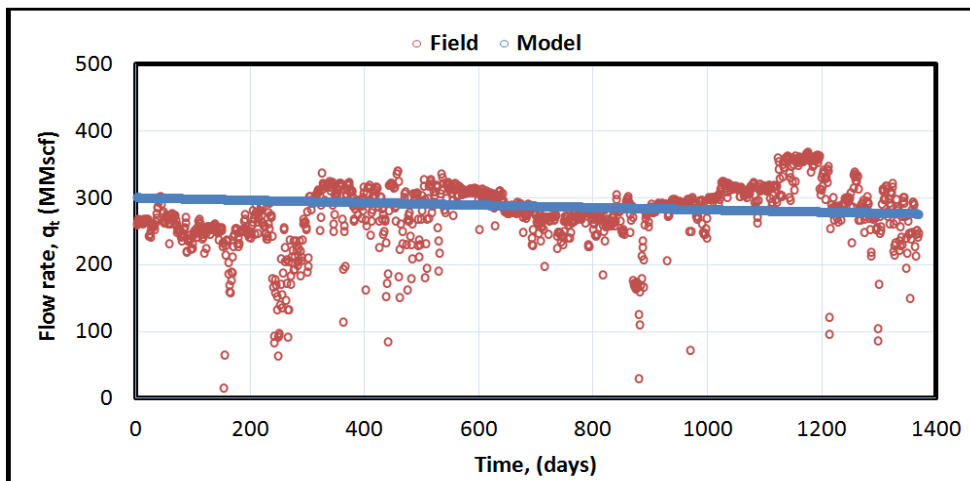


Fig. 5. Gas production rate - Time plot (Hyperbolic model)

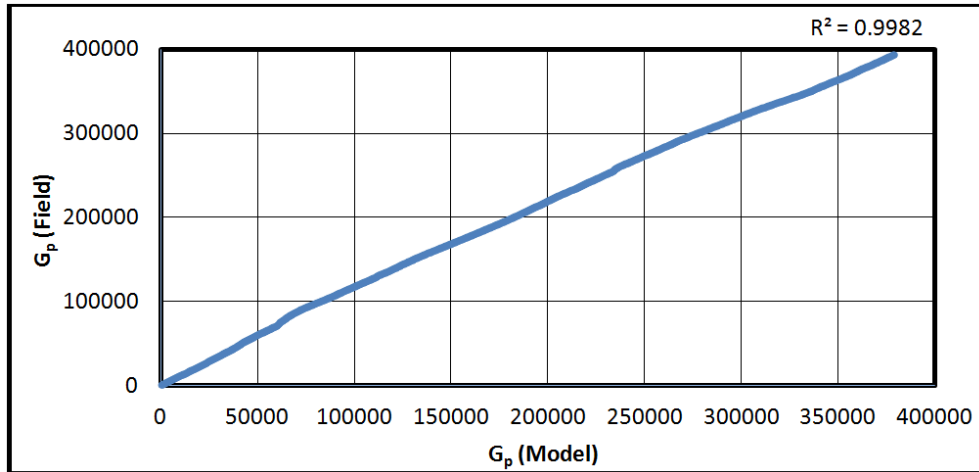


Fig. 6. Field data – Model predicted cumulative production plot (Hyperbolic model)

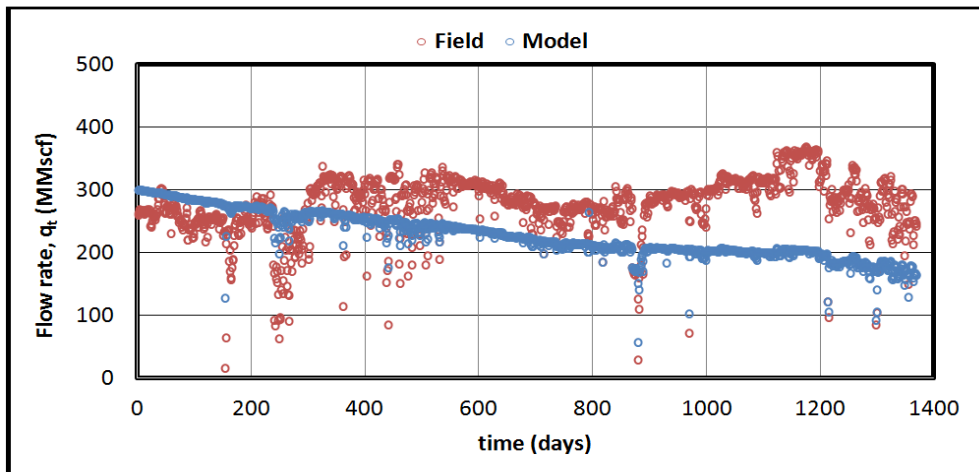


Fig. 7. Gas production rate - Time plot (Reciprocal model)

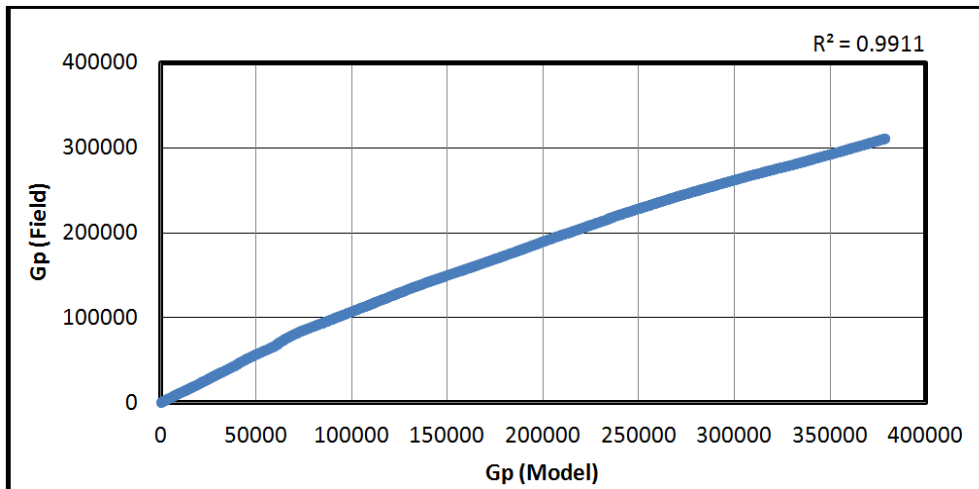


Fig. 8. Field data – Model predicted cumulative production plot (Reciprocal model)

### 3.3 Quadratic Model

Figs. 9 and 10 present the fitted Quadratic model predictions. The former Figure is the comparison of the predicted production rate ( $q_{t_{model}}$ ) with the actual field production rate ( $q_{t_{field}}$ ). Observation shows that this fitted model's predicted production rate aligned closely with the actual field data than the other models (i.e., Arps' and Reciprocal model). This efficient alignment of the production rate resulted in excellent matching of the predicted cumulative gas production with the actual field data, as depicted in Figs. 9 and B-5 (in Appendix B). Furthermore, the result shows that the predicted cumulative gas production ( $G_{P_{model}}$ ) has a coefficient of determination ( $r^2$ ) of

0.9998 with the actual field cumulative gas production ( $G_{P_{field}}$ ).

Finally, the fitted models' predictions are close to the actual field production data. However, a comparison of all the fitted models predictions; as depicted in Figs. B-6 through B-8, indicate that the Arps' and Quadratic models have close predictions, even with the actual field data. But the Reciprocal model predictions are close to the actual field production data and other models at the early period of production. Therefore, the fitted Quadratic model can be used as a quick tool to predict the performance of "ABC" gas field in the Niger Delta region.

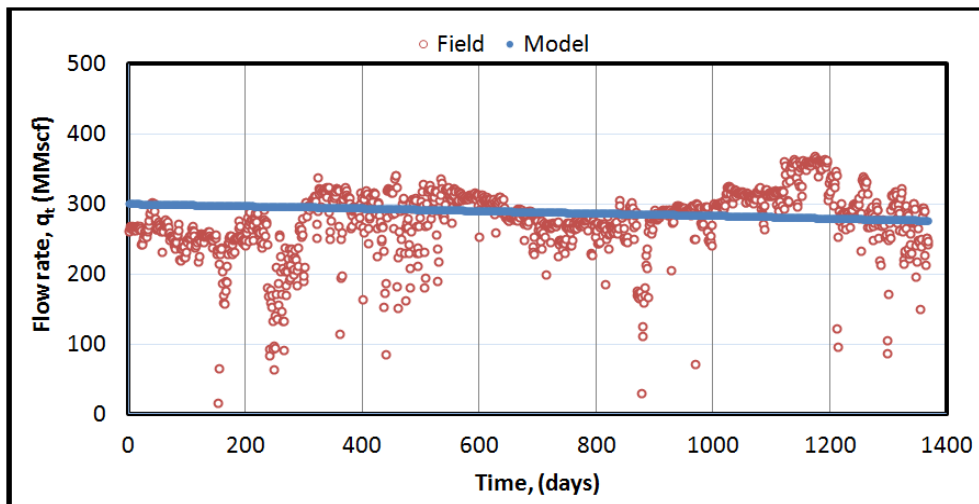


Fig. 9. Flow rate - Time plot (Quadratic model)

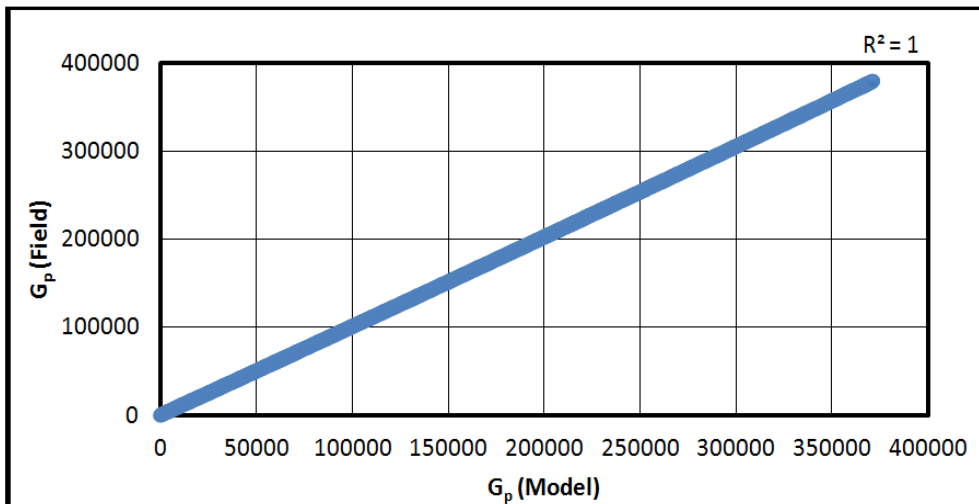


Fig. 10. Field data – Model predicted cumulative production plot (Quadratic model)



#### 4. CONCLUSION

Qualitative research in the Petroleum Industry in Nigeria has largely focused on oil reservoirs with little or no recourse to gas reservoirs. With the paradigm which is beginning to tilt towards gas production, brings to bare the necessity and peculiarity with respect to gas production and its challenges. This paper is based on the concept of rate decline approach by comparing Arps' models with two recently proposed models: Reciprocal and Quadratic model to assess their prediction of gas reservoir performance in the Niger Delta. These fitted models were tested and validated with field production data and the following conclusions can be made:

- i. The established decline constant ( $D_i$ ) for the 'ABC' gas field in Niger Delta is  $0.000064\text{day}^{-1}$ ;
- ii. The Arps' models - Exponential and Harmonic are sufficient to predict the 'ABC' gas field performance with an absolute error, standard deviation and coefficient of determination of 0.1150, 0.02666 and 0.9981, and 0.11547, 0.02665 and 0.9982 respectively;
- iii. The Reciprocal model prediction of the gas field performance is relatively accurate in the early period of production with an absolute error, standard deviation and coefficient of determination of 0.09726, 0.026745 and 0.9911 respectively; and
- iv. The Quadratic model accurately predicted the field production data with an absolute error, standard deviation and coefficient of determination of 0.0097, 0.000008 and 0.9998, respectively. Therefore, the model can be used as a quick tool to predict gas field 'ABC' performance in the Niger Delta region.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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## APPENDIX A

The equations used for the statistical analysis of the fitted models' prediction and field test data:

1. Average Error:

$$E_{avg} = \frac{1}{N} \sum_{i=1}^n \frac{G_{P_{field}} - G_{P_{model}}}{G_{P_{field}}} \quad (A - 1)$$

2. Absolute Error:

$$E_{abs} = \frac{1}{N} \sum_{i=1}^n \left| \frac{G_{P_{field}} - G_{P_{model}}}{G_{P_{field}}} \right| \quad (A - 2)$$

3. Root Mean Square Error:

$$E_{rms} = \frac{1}{N} \sum_{i=1}^n \left( \frac{G_{P_{field}} - G_{P_{model}}}{G_{P_{field}}} \right)^2 \quad (A - 3)$$

4. Normalized Root mean Square Error:

$$E_{nrms} = \frac{E_{rms}}{G_{P_{field}(\max)} - G_{P_{field}(\min)}} \quad (A - 4)$$

5. Coefficient of Determination:

$$r^2 = 1 - \frac{\sum (G_{P_{field}} - G_{P_{model}})^2}{\sum (G_{P_{field}} - \bar{G}_{P_{model}})^2} \quad (A - 5)$$

6. Standard Deviation:

$$SD = \frac{1}{N} \sqrt{\sum_{i=1}^n \left( \frac{G_{P_{field}} - G_{P_{model}}}{G_{P_{field}}} \right)^2 - \sum_{i=1}^n \left( \frac{G_{P_{field}} - \bar{G}_{P_{model}}}{G_{P_{field}}} \right)^2} \quad (A - 6)$$

7. Normalized Standard Deviation

$$NSD = 100 \sqrt{\frac{1}{N-1} \sum_{i=0}^n \left( \frac{G_{P_{field}} - G_{P_{model}}}{G_{P_{field}}} \right)^2} \quad (A - 7)$$

where:

$G_{p_{field}}$  = Field Data

$G_{p_{model}}$  = Predicted Data

$N$  = Number Field Data

$\bar{G}_{p_{model}}$  = Average Predicted Data

### APPENDIX B

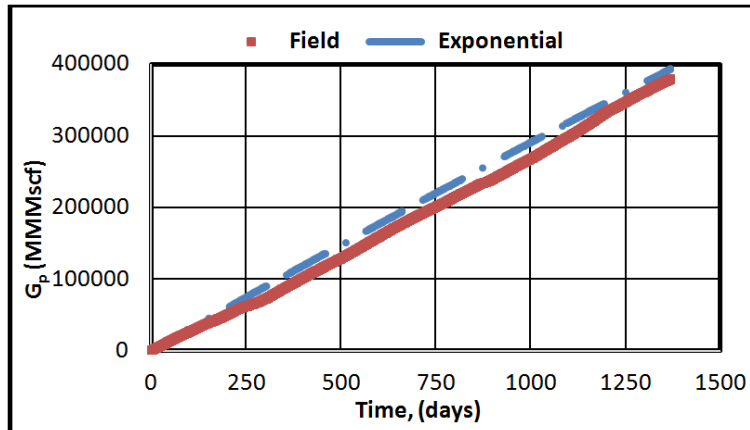


Fig. B-1. Cumulative production - Time plot (Exponential model)

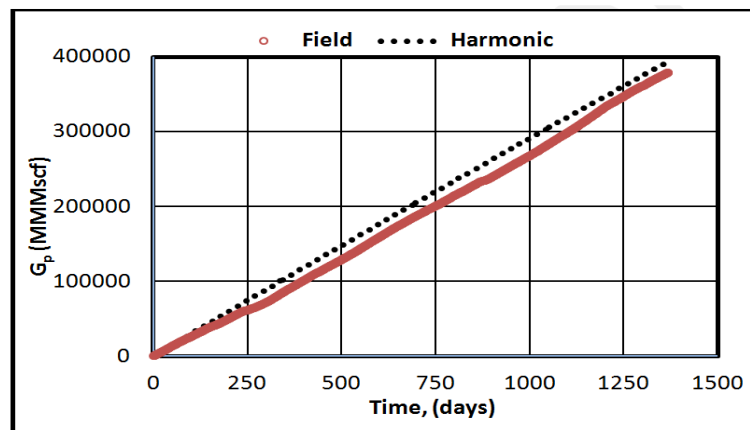


Fig. B-2. Cumulative production vs Time plot (Harmonic model)

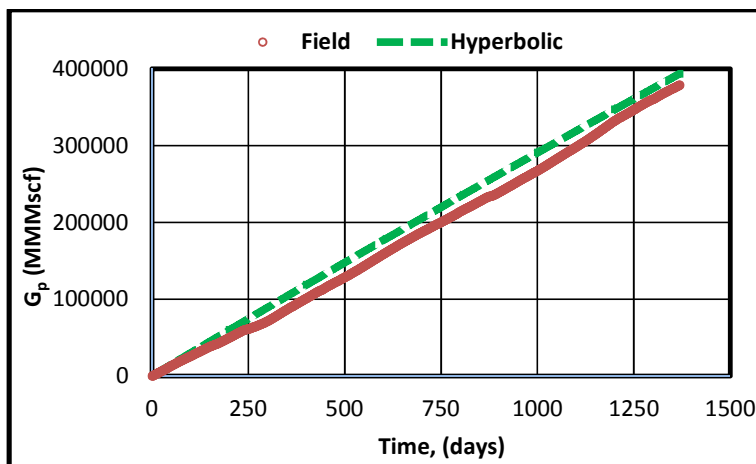


Fig. B-3. Cumulative production - Time plot (Hyperbolic model)

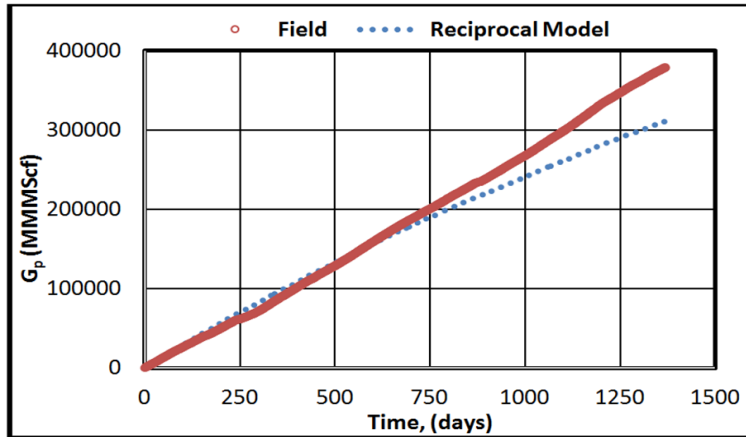


Fig. B-4. Cumulative production - Time plot (Reciprocal model)

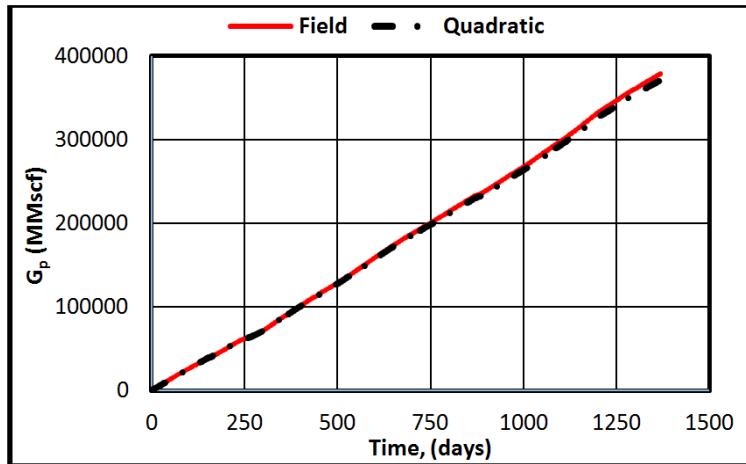


Fig. B-5. Cumulative production - Time plot (Quadratic model)

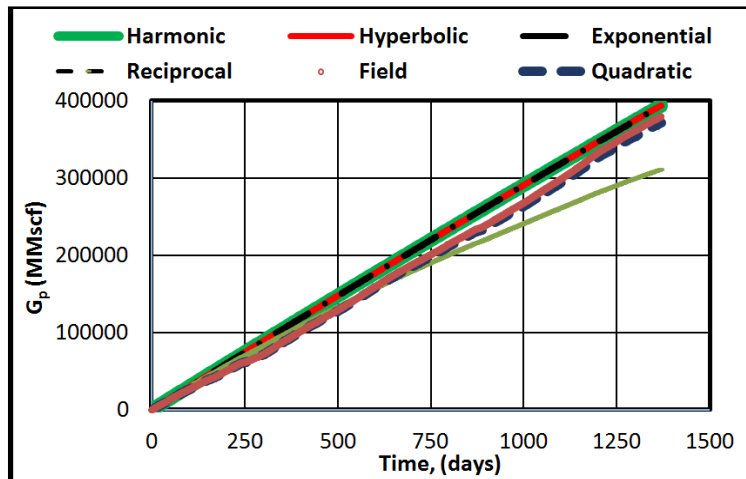


Fig. B-6. Cumulative production - Time plot (All models)

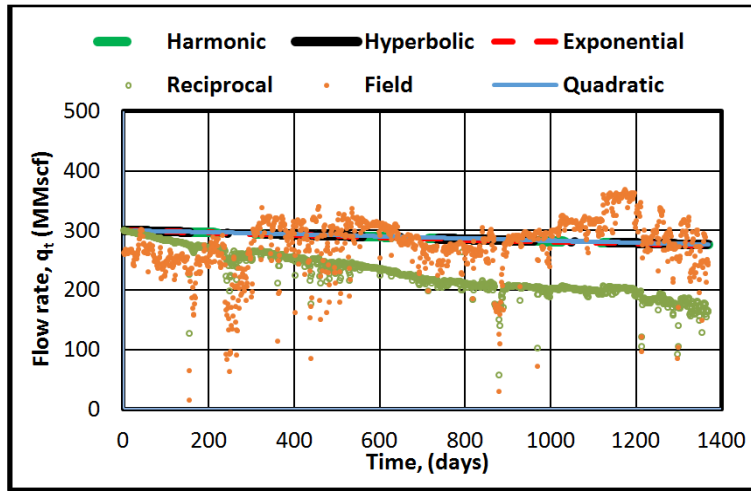


Fig. B-7. Gas production rate - Time plot (All models)

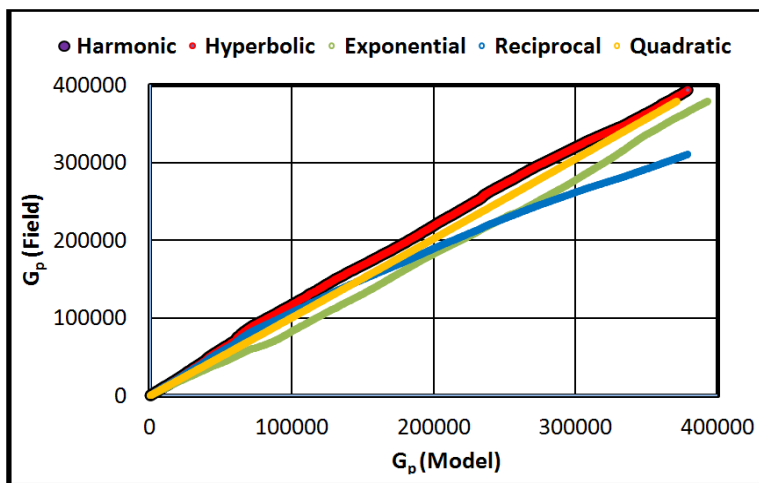


Fig. B-8. Field data – Model predicted cumulative production plot (All models)

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