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# On A Shape Parameter of Gompertz Inverse Exponential Distribution Using Classical and Non Classical Methods of Estimation

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***Authors' contributions***

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## ABSTRACT

The Gompertz inverse exponential distribution is a three-parameter lifetime model with greater flexibility and performance for analyzing real life data. It has one scale parameter and two shape parameters responsible for the flexibility of the distribution. Despite the importance and necessity of parameter estimation in model fitting and application, it has not been established that a particular estimation method is better for any of these three parameters of the Gompertz inverse exponential distribution. This article focuses on the development of Bayesian estimators for a shape of the Gompertz inverse exponential distribution using two non-informative prior distributions (Jeffery and Uniform) and one informative prior distribution (Gamma prior) under Square error loss function (SELF), Quadratic loss function (QLF) and Precautionary loss function (PLF). These results are compared with the maximum likelihood counterpart using Monte Carlo simulations. Our results

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indicate that Bayesian estimators under Quadratic loss function (QLF) with any of the three prior distributions provide the smallest mean square error for all sample sizes and different values of parameters.

*Keywords: Bayesian method; uniform prior; Jeffrey's prior; gamma prior; loss functions; MLE; MSE; sample sizes.*

## 1. INTRODUCTION

The The Gompertz inverse exponential distribution (GoIED) is an extension of the inverse exponential distribution recently proposed by Oguntunde et al. [1] with many properties of the model studied and discussed accordingly. The probability density function (pdf) of the model and its failure rate have unimodal shapes which is an indication that the model would be useful for real-life events with unimodal failure rates. This distribution is found to be tractable and flexible and shows high modeling capability due to the fact that it outperforms other important models such as the Gompertz exponential distribution, Gompertz Weibull distribution and Gompertz Lomax distribution after some applications to real life data [1]. The GoIED is a very competitive model, and it is hoped that it would be of use in fields like engineering, biology and medicine. Considering the importance of this distribution, it is of no doubt very necessary to identify the best methods for estimating the parameters of the GoIED which will remain useful in all possible applications of this model in most real life situations as mentioned earlier.

According to Oguntunde, et al. [1], the probability density function (pdf), the cumulative distribution function (cdf), the survival function (sf), the hazard function (or failure rate) and quantile function (qf) of the GoIED are respectively defined as:

$$f(x) = \frac{\alpha\theta}{x^2} e^{-\frac{\theta}{x}} \left[1 - e^{-\frac{\theta}{x}}\right]^{-\beta-1} e^{\frac{\alpha}{\beta}\left[1-\left(1-e^{-\frac{\theta}{x}}\right)^{-\beta}\right]} \quad (1)$$

$$F(x) = 1 - e^{\frac{\alpha}{\beta}\left[1-\left(1-e^{-\frac{\theta}{x}}\right)^{-\beta}\right]} \quad (2)$$

$$S(x) = 1 - F(x) = e^{\frac{\alpha}{\beta}\left[1-\left(1-e^{-\frac{\theta}{x}}\right)^{-\beta}\right]} \quad (3)$$

$$h(x) = \frac{f(x)}{S(x)} = \frac{\alpha\theta}{x^2} e^{-\frac{\theta}{x}} \left[1 - e^{-\frac{\theta}{x}}\right]^{-\beta-1} \quad (4)$$

and

$$Q(u) = \theta \log\left\{1 - \left(1 - \frac{\beta\log(1-u)}{\alpha}\right)^{-\frac{1}{\beta}}\right\} \quad (5)$$

For $x > 0, \alpha, \beta, \theta > 0$ where $\alpha$ *and* $\beta$ are the shape parameters and $\theta$ is the scale parameter of the distribution.

These functions are represented graphically using some arbitrary parameter values as displayed in the following figures.

In statistics, we have two basic methods of parameter estimation and these are the classical and the non classical methods. In the classical theory of estimation, the parameters are taken to be fixed but unknown whereas we consider the parameters to be unknown and random just like variables under non classical method. The most popular and unique method under classical theory is the method of maximum likelihood estimation while the Bayesian estimation method is considered under non classical theory. But, in common real life problems described by life time distributions, the parameters cannot be treated as fixed in all the life testing period according to Martz and Waller [2] as well as Ibrahim et al. [3] and Singpurwalla [4]. Based on this fact, it becomes obvious the frequentist or classical approach can no longer handle adequately problems of parameter estimation in life time models and therefore the need for non classical or Bayesian estimation in life time models.

Due to the stated problem above, a number of research works on Bayesian estimation method of parameters have been conducted and a highlight of some of these studies which dependent on the distribution in question are as follows: Bayesian estimation for the extreme value distribution using progressive censored data and asymmetric loss by Al-Aboud [5], Bayesian estimators of the shape and scale parameters of modified Weibull distribution using Lindley's approximation under the squared error loss function, LINEX loss function and generalized entropy loss function by Preda et al. [6], comparison of Bayesian estimates of the shape parameter of Generalized Exponential Distribution based on a class of non-informative prior under the assumption of quadratic loss function, squared log-error loss function and general entropy loss function (*GELF*) and maximum likelihood estimates by Dey [7], Bayesian Survival Estimator for Weibull distribution with censored data by Ahmed et al. [8] as well as Pandey et al. [9], Al-Athari [10]. Ieren and Yahaya [11] introduced and studied the properties of a Weimal distribution with applications to real life data while Yahaya and Ieren [12] estimated its parameters using maximum likelihood method which was in recent times compared with the Bayesian approach considering a scale parameter of the Weimal distribution by Mabur et al. [13] in a study titled "Bayesian Estimation of the Scale Parameter of the Weimal Distribution" in which the results show that Bayesian estimators of the scale parameter were better than that of maximum likelihood method. Similarly, Aliyu and Yahaya [14] studied the shape parameter of generalized Rayleigh distribution under non-informative priors with a comparison to the method of maximum likelihood. Also, a good number of loss functions have been shown to have performed better during estimation under Bayesian method in so many studies including "Bayesian analysis of Weibull distribution using R software" by Ahmad and Ahmad [15], "On parameter estimation of erlang distribution using Bayesian method under different loss functions" by Ahmad et al. [16], "Classical and Bayesian approach in estimation of the scale parameter of Nakagami distribution"

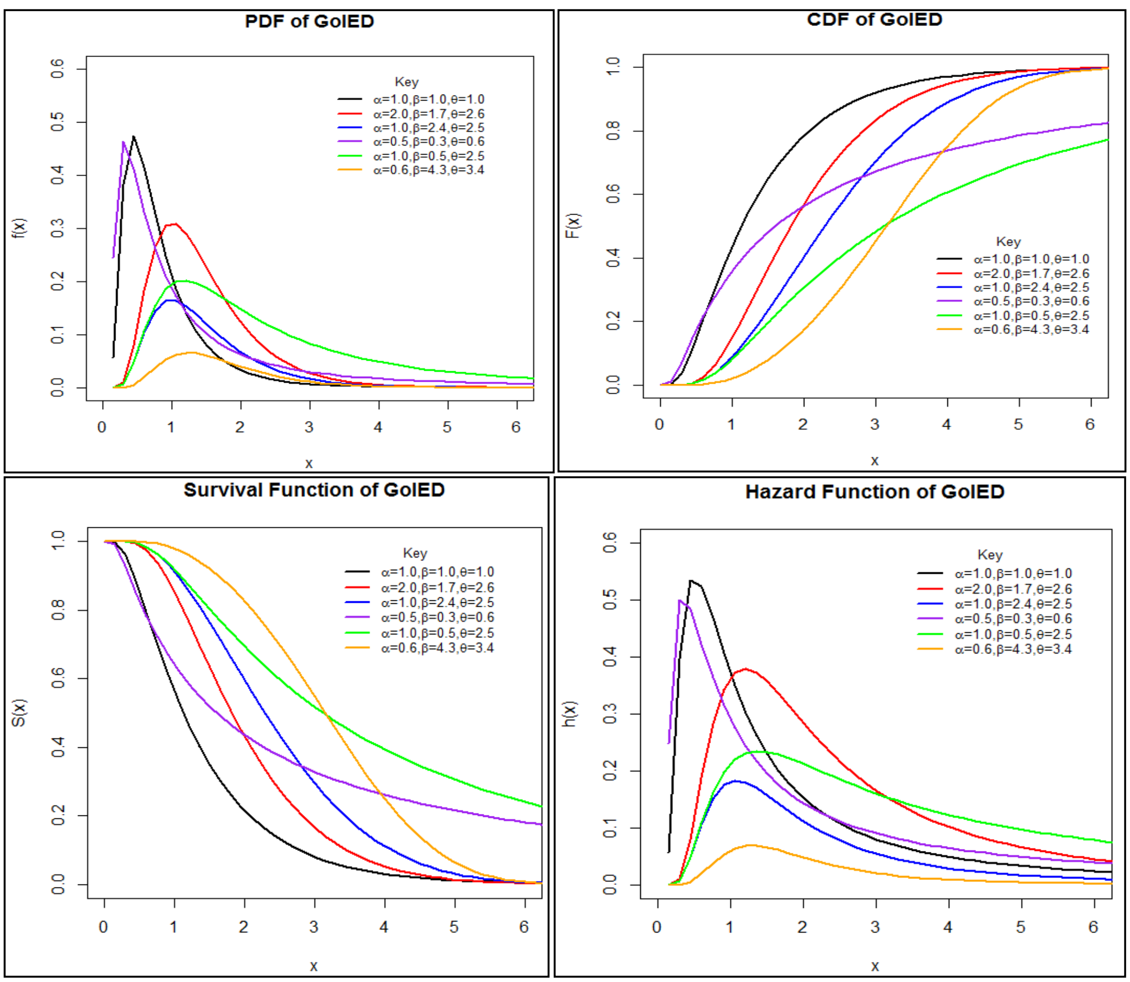


**Fig. 1. Plots of the PDF, CDF, survival function and hazard function of the GoIED for selected parameter values**

by Ahmad et al. [17], "Maximum Likelihood and Bayes Estimation in Randomly Censored Geometric Distribution" by Krishna and Goel [18], "a comparison between maximum likelihood and Bayesian estimation methods for a shape parameter of the Weibull-exponential distribution" by Ieren and Oguntunde [19], "classical and Bayesian estimation of Weibull distribution in the presence of outliers" by Gupta and Singh [20], "Estimation of parameter and reliability function of the exponentiated inverted Weibull distribution using classical and Bayesian approach" by Gupta [21], "Bayesian estimation of a shape parameter of the Weibull-Frechet distribution" by Ieren and Chukwu [22], "posterior properties of the Nakagami-m distributions using non-informative priors and applications in reliability" by Ramos et al. [23], "Bayesian reference analysis for the generalized gamma distribution" by Ramos and Louzada [24] and "Reference Bayesian analysis for the generalized lognormal distribution with application to survival data" by Tomazella et al. [25] e.t.c.

The method of estimating parameters in a distribution differs from one parameter of a distribution to another and the Gompertz inverse exponential distribution is not an exception. Hence, this study aims at estimating one shape parameter of the Gompertz inverse exponential distribution using Bayesian approach and making a comparison between the Bayesian approach and the method of maximum likelihood estimation.

The aim of this article is to estimate a shape parameter of the Gompertz inverse exponential distribution with Bayesian method using uniform prior, Jeffrey's prior and gamma prior with three loss functions for the case of complete samples. Including this introductory section, the rest of this article unfolds as follows: in Section 2, maximum likelihood estimator (MLE) for the shape parameter is obtained. In Section 3, Bayesian estimators based on different loss functions by taking uniform prior, Jeffrey's prior and gamma prior are derived. The proposed estimators are compared in terms of their mean squared error (MSE) in Section 4. Finally, conclusions and recommendations are presented in Section 5.

## 2. MAXIMUM LIKELIHOOD ESTIMATION

Maximum Likelihood is a popular estimation technique for many distributions because it picks the values of the distribution's parameters that make the data more likely" than any other value of the parameter. This is accomplished by maximizing the likelihood function of the parameter(s) given the data.

Let $x_1, x_2, ......, x_n$ be a random sample from a population $X$ with probability density function $f(x)$, . The likelihood is the joint probability function of the data, but viewed as a function of the parameters, treating the observed data as fixed quantities. Assuming that the values, $\underline{x} = (x_1, x_2, ..., x_n)$ are obtained independently, the likelihood function is given by

$$L(\underline{x} \mid \theta) = P(x_1, x_2, ..., x_n \mid \theta) = \prod_{i=1}^{n} P(\underline{x} \mid \theta) \tag{6}$$

The likelihood function, $L(\underline{x} \mid \alpha, \beta, \theta)$, is defined to be the joint density of the random variables $x_1, x_2, ......, x_n$ and it is given as

$$L(X \mid \alpha, \beta, \theta) \propto (\alpha\theta)^n \prod_{i=1}^{n} \left( x^{-2} \mathrm{e}^{-\frac{\theta}{x}} \left[ 1 - \mathrm{e}^{-\frac{\theta}{x_i}} \right]^{-\beta-1} \right) \mathrm{e}^{\frac{\alpha}{\beta} \sum_{i=1}^{n} \left[ 1 - \left( 1 - e^{-\frac{\theta}{x_i}} \right)^{-\beta} \right]} \tag{7}$$

The likelihood function for the shape parameter, $\alpha$ , is given by;

$$L(\underline{x} \mid \alpha) \propto \alpha^n \mathrm{e}^{\frac{\alpha}{\beta} \sum_{i=1}^{n} \left[ 1 - \left( 1 - e^{-\frac{\theta}{x_i}} \right)^{-\beta} \right]} = K\alpha^n \mathrm{e}^{\frac{\alpha}{\beta} M} \tag{8}$$

Where $M = \sum_{i=1}^{n} \left[ 1 - \left( 1 - e^{-\frac{\theta}{x_i}} \right)^{-\beta} \right]$ and $K = \theta^n \prod_{i=1}^{n} \left( x^{-2} \mathrm{e}^{-\frac{\theta}{x}} \left[ 1 - \mathrm{e}^{-\frac{\theta}{x_i}} \right]^{-\beta-1} \right)$ are constants all of which are independent of the shape parameter, $\alpha$ .

Let the log-likelihood function, $l = \log L(\underline{x} \mid \alpha)$, therefore

$$l = \log L(\underline{x} \mid \alpha) = \log K + n \log \alpha + \frac{\alpha}{\beta} M \tag{9}$$

Differentiating $l$ partially with respect to $\alpha$, the shape parameter and solving for $\hat{\alpha}$ gives;

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \frac{1}{\beta}M = 0 \quad \Rightarrow \hat{\alpha} = \frac{-\beta n}{M} \tag{10}$$

where $\hat{\alpha}$ is the maximum likelihood estimator of the shape parameter, $\alpha$.

## 3. BAYESIAN ESTIMATION

The Bayesian inference requires appropriate choice of prior(s) for the parameter(s). From the Bayesian viewpoint, there is no clear cut way from which one can conclude that one prior is better than the other. Nevertheless, very often priors are chosen according to one's subjective knowledge and beliefs. However, if one has adequate information about the parameter(s), it is better to choose informative prior(s); otherwise, it is preferable to use non-informative prior(s).

To obtain the posterior distribution of the shape parameter once the data has been observed, we apply bayes' Theorem which is stated in the following form:

$$P(\alpha \mid \underline{x}) = \frac{P(\alpha, \underline{x})}{P(\underline{x})} = \frac{P(\underline{x} \mid \alpha)P(\alpha)}{P(\underline{x})} = \frac{P(\underline{x} \mid \alpha)P(\alpha)}{\int P(\underline{x} \mid \alpha)P(\alpha)d\alpha}$$
$$= \frac{L(\underline{x} \mid \alpha)P(\alpha)}{\int L(\underline{x} \mid \alpha)P(\alpha)d\alpha} \tag{11}$$

Where $P(\underline{x})$ is the marginal distribution of X and $P(\underline{x}) = \sum_{\infty} p(\alpha)L(\underline{x} \mid \alpha)$ when the prior distribution of $\alpha$ is discrete and $P(\underline{x}) = \int_{-\infty}^{\infty} p(\alpha)L(\underline{x} \mid \alpha)d\alpha$ when the prior distribution of $\alpha$ is continuous. Also note that $p(\alpha)$ and $L(\underline{x} \mid \alpha)$ are the prior distribution and the Likelihood function respectively.

## 4. BAYESIAN ANALYSIS UNDER THE ASSUMPTION OF UNIFORM PRIOR USING THREE LOSS FUNCTIONS

The uniform prior is defined as:

$$p(\alpha) \propto 1; 0 < \alpha < \infty \tag{12}$$

The posterior distribution of the shape parameter $\alpha$ under uniform prior is obtained from equation (11) using integration by substitution method as

$$P(\alpha \mid \underline{x}) = \frac{M^{n+1}}{(-\beta)^{n+1}\Gamma(n+1)}\alpha^n e^{\frac{\alpha}{\beta}M} \tag{13}$$

With the above uniform prior and posterior distribution from it, we will use three loss functions to estimate the shape parameter of the *GoIED* and these loss functions are defined as follows:

**(a) Squared Error Loss Function (SELF)**

The squared error loss function proposed by Legendre (1805) and Gauss (1810) relating to the shape parameter $\alpha$ is defined as

$$L(\alpha, \alpha_{SELF}) = (\alpha - \alpha_{SELF})^2 \tag{14}$$

where $\alpha_{SELF}$ is the estimator of the parameter $\alpha$ under *SELF*.

**(b) Quadratic Loss Function (QLF)**

The quadratic loss function is defined from Azam and Ahmad [26] as

$$L(\alpha, \alpha_{QLF}) = \left(\frac{\alpha - \alpha_{QLF}}{\alpha}\right)^2 \tag{15}$$

where $\alpha_{QLF}$ is the estimator of the parameter $\alpha$ under *QLF*.

**(c) Precautionary Loss Function (PLF)**

The precautionary loss function (*PLF*) introduced by Norstrom [27] is an asymmetric loss function and is defined as

$$L(\alpha_{PLF}, \alpha) = \frac{(\alpha_{PLF} - \alpha)^2}{\alpha_{PLF}} \tag{16}$$

where $\alpha_{PLF}$ is the estimator of the shape parameter $\alpha$ under *PLF*.

Now the Bayes estimators under uniform prior using *SELF*, *QLF* and *PLF* are given respectively as follows:

$$\alpha_{SELF} = E(\alpha) = E(\alpha \mid \underline{x}) = \int_0^{\infty} \alpha P(\alpha \mid \underline{x})d\alpha = \frac{-\beta(n+1)}{M} \tag{17}$$

$$\alpha_{QLF} = \frac{E(\alpha^{-1} \mid \underline{x})}{E(\alpha^{-2} \mid \underline{x})} = \frac{\int_0^{\infty} \alpha^{-1}P(\alpha \mid \underline{x})d\alpha}{\int_0^{\infty} \alpha^{-2}P(\alpha \mid \underline{x})d\alpha} = \frac{-\beta(n-1)}{M} \tag{18}$$

and

$$\alpha_{PLF} = \left\{E\left(\alpha^2 \mid \underline{x}\right)\right\}^{\frac{1}{2}} = \left\{\int_0^\infty \alpha^2 P\left(\alpha \mid \underline{x}\right) d\alpha\right\}^{\frac{1}{2}} \quad (19)$$

$$= \frac{-\beta\left[(n+1)(n+2)\right]^{0.5}}{M}$$

## 5. BAYESIAN ANALYSIS UNDER THE ASSUMPTION OF JEFFREY'S PRIOR USING THREE LOSS FUNCTIONS

Also, the Jeffrey's prior is defined as:

$$p(\alpha) \propto \frac{1}{\alpha}; 0 < \alpha < \infty \quad (20)$$

The posterior distribution of the shape parameter $\alpha$ for a given data under Jeffrey prior is obtained from equation (11) using integration by substitution method as

$$P\left(\alpha \mid \underline{x}\right) = \frac{M^n}{(-\beta)^n \Gamma(n)} \alpha^{n-1} e^{\frac{\alpha}{\beta}M} \quad (21)$$

Again the Bayes estimators under Jeffrey's prior using *SELF*, *QLF* and *PLF* are given respectively as follows:

$$\alpha_{SELF} = E(\alpha) = E\left(\alpha \mid \underline{x}\right) = \int_0^\infty \alpha P\left(\alpha \mid \underline{x}\right) d\alpha = \frac{-\beta(n)}{M} \quad (22)$$

$$\alpha_{QLF} = \frac{E\left(\alpha^{-1} \mid \underline{x}\right)}{E\left(\alpha^{-2} \mid \underline{x}\right)} = \frac{\int_0^\infty \alpha^{-1} P\left(\alpha \mid \underline{x}\right) d\alpha}{\int_0^\infty \alpha^{-2} P\left(\alpha \mid \underline{x}\right) d\alpha} = \frac{-\beta(n-2)}{M} \quad (23)$$

and

$$\alpha_{PLF} = \left\{E\left(\alpha^2 \mid \underline{x}\right)\right\}^{\frac{1}{2}} = \left\{\int_0^\infty \alpha^2 P\left(\alpha \mid \underline{x}\right) d\alpha\right\}^{\frac{1}{2}} \quad (24)$$

$$= \frac{-\beta\left[n(n+1)\right]^{0.5}}{M}$$

## 6. BAYESIAN ANALYSIS UNDER THE ASSUMPTION OF GAMMA PRIOR USING THREE LOSS FUNCTIONS

Also, the gamma prior is defined as:

$$P(\alpha) = \frac{a^b}{\Gamma(b)} \alpha^{b-1} e^{-a\alpha} \quad (25)$$

The posterior distribution of the shape parameter $\alpha$ for a given data under gamma prior is obtained from equation (11) using integration by substitution method as

$$P\left(\alpha \mid \underline{x}\right) = \frac{\left(a - \frac{M}{\beta}\right)^{n+b}}{(-\beta)^n \Gamma(n+b)} \alpha^{n+b-1} e^{\alpha\left(a - \frac{M}{\beta}\right)} \quad (26)$$

Also the Bayes estimators under gamma prior using *SELF*, *QLF* and *PLF* are given respectively as:

$$\alpha_{SELF} = \frac{n+b}{a - \frac{M}{\beta}} \quad (27)$$

$$\alpha_{QLF} = \frac{n+b-2}{a - \frac{M}{\beta}} \quad (28)$$

and

$$\alpha_{PLF} = \frac{\left[(n+b+1)(n+b)\right]^{0.5}}{a - \frac{M}{\beta}} \quad (29)$$

## 7. RESULTS AND DISCUSSION

In this section, Monte Carlo simulation with R software under 10,000 replications is considered to generate random samples of sizes n = (10, 25, 50, 100, 150, 200) from Gompertz inverse exponential distribution using the quantile function (inverse transformation method of simulation) under the following combination of parameter values: $\alpha = 0.5, \beta = 0.5, \theta = 0.5, a = 1$ and $b = 2$; $\alpha = 0.5, \beta = 0.5, \theta = 2, a = 1$ and $b = 2$; $\alpha = 0.5, \beta = 2, \theta = 0.5, a = 1$ and $b = 2$ and $\alpha = 2, \beta = 0.5, \theta = 0.5, a = 1$ and $b = 2$. The following tables present the results of our simulation study by listing the average estimates of the shape parameter with their respective Mean Square Errors (MSEs) under the appropriate estimation methods which include the Maximum Likelihood Estimation (*MLE*), Squared Error Loss Function (*SELF*), Quadratic Loss Function (*QLF*), and Precautionary Loss Function (*PLF*) under Uniform Jeffrey and gamma priors respectively. The criterion for evaluating the performance of the estimators in this study is the Mean Square Error (MSE): $MSE = \frac{1}{n} E\left(\hat{\alpha} - \alpha\right)^2$.

The results in Tables 1-4 show that the estimator of the shape parameter using QLF under Gamma, uniform and Jeffrey priors is better than the other estimators with small MSE irrespective of the variation in the samples. This behavior of minimum MSE for Bayesian estimation (using

QLF under Uniform, Jeffrey and gamma priors) is an indication that the method is the most efficient for estimating this shape parameter compared to the Method of Maximum Likelihood estimation (*MLE*) and Bayesian with other loss functions. More so, comparing the QLF under the prior distributions it is discovered that the QLF under the Jeffrey prior has the smallest MSE compared to the QLF under uniform and gamma priors for a smaller value of the shape parameter whereas, the QLF under the gamma prior has the smallest MSE compared to the QLF under uniform and

**Table 1. Average estimates (Estimates) and Mean Squared Errors (MSEs) for $\alpha = 0.5, \beta = 0.5, \theta = 0.5, a = 1$ and $b = 2$ under different priors, loss functions and sample sizes**

| n | Measures | MLE | Uniform prior | | | Jeffrey's prior | | | Gamma prior | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| **10** | Estimate | 0.5579 | 0.6137 | 0.5021 | 0.6409 | 0.5579 | 0.4463 | 0.5851 | 0.6303 | 0.5252 | 0.6560 |
| | MSE | 0.0415 | 0.0591 | 0.0309 | 0.0702 | 0.0415 | 0.0273 | 0.0492 | 0.0588 | 0.0297 | 0.0697 |
| **25** | Estimate | 0.5197 | 0.5404 | 0.4989 | 0.5507 | 0.5197 | 0.4781 | 0.530 | 0.5493 | 0.5086 | 0.5594 |
| | MSE | 0.0120 | 0.0142 | 0.0107 | 0.0156 | 0.0120 | 0.0103 | 0.013 | 0.0148 | 0.0107 | 0.0164 |
| **50** | Estimate | 0.5106 | 0.5208 | 0.5004 | 0.5259 | 0.5106 | 0.4902 | 0.5157 | 0.5255 | 0.5053 | 0.5306 |
| | MSE | 0.0056 | 0.0061 | 0.0052 | 0.0065 | 0.0056 | 0.0051 | 0.0058 | 0.0063 | 0.0053 | 0.0067 |
| **100** | Estimate | 0.5047 | 0.5098 | 0.4997 | 0.5123 | 0.5047 | 0.4946 | 0.5072 | 0.5122 | 0.5022 | 0.5147 |
| | MSE | 0.0027 | 0.0028 | 0.0026 | 0.0029 | 0.0027 | 0.0026 | 0.0027 | 0.0029 | 0.0026 | 0.0030 |
| **150** | Estimate | 0.5022 | 0.5056 | 0.4989 | 0.5072 | 0.5022 | 0.4955 | 0.5039 | 0.5072 | 0.5005 | 0.5089 |
| | MSE | 0.0017 | 0.0018 | 0.0017 | 0.0018 | 0.0017 | 0.0017 | 0.0018 | 0.0018 | 0.0017 | 0.0019 |
| **200** | Estimate | 0.5026 | 0.5051 | 0.5001 | 0.5064 | 0.5026 | 0.4976 | 0.5039 | 0.5063 | 0.5013 | 0.5076 |
| | MSE | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0013 | 0.0013 | 0.0013 | 0.0013 |

**Table 2. Average estimates (Estimates) and Mean Squared Errors (MSEs) for $\alpha = 0.5, \beta = 0.5, \theta = 2, a = 1$ and $b = 2$ under different priors, loss functions and sample sizes**

| n | Measures | MLE | Uniform prior | | | Jeffrey's prior | | | Gamma prior | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| **10** | Estimate | 0.5579 | 0.6137 | 0.5021 | 0.6409 | 0.5579 | 0.4463 | 0.5851 | 0.6303 | 0.5252 | 0.6560 |
| | MSE | 0.0415 | 0.0591 | 0.0309 | 0.0702 | 0.0415 | 0.0273 | 0.0492 | 0.0588 | 0.0297 | 0.0697 |
| **25** | Estimate | 0.5197 | 0.5404 | 0.4989 | 0.5507 | 0.5197 | 0.4781 | 0.530 | 0.5493 | 0.5086 | 0.5594 |
| | MSE | 0.0120 | 0.0142 | 0.0107 | 0.0156 | 0.0120 | 0.0103 | 0.013 | 0.0148 | 0.0107 | 0.0164 |
| **50** | Estimate | 0.5106 | 0.5208 | 0.5004 | 0.5259 | 0.5106 | 0.4902 | 0.5157 | 0.5255 | 0.5053 | 0.5306 |
| | MSE | 0.0056 | 0.0061 | 0.0052 | 0.0065 | 0.0056 | 0.0051 | 0.0058 | 0.0063 | 0.0053 | 0.0067 |
| **100** | Estimate | 0.5047 | 0.5098 | 0.4997 | 0.5123 | 0.5047 | 0.4946 | 0.5072 | 0.5122 | 0.5022 | 0.5147 |
| | MSE | 0.0027 | 0.0028 | 0.0026 | 0.0029 | 0.0027 | 0.0026 | 0.0027 | 0.0029 | 0.0026 | 0.0030 |
| **150** | Estimate | 0.5022 | 0.5056 | 0.4989 | 0.5072 | 0.5022 | 0.4955 | 0.5039 | 0.5072 | 0.5005 | 0.5089 |
| | MSE | 0.0017 | 0.0018 | 0.0017 | 0.0018 | 0.0017 | 0.0017 | 0.0018 | 0.0018 | 0.0017 | 0.0019 |
| **200** | Estimate | 0.5026 | 0.5051 | 0.5001 | 0.5064 | 0.5026 | 0.4976 | 0.5039 | 0.5063 | 0.5013 | 0.5076 |
| | MSE | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0013 | 0.0013 | 0.0013 | 0.0013 |

**Table 3. Average estimates (Estimates) and Mean Squared Errors (MSEs) for $\alpha = 0.5, \beta = 2, \theta = 0.5, a = 1$ and $b = 2$ under different priors, loss functions and sample sizes**

| n | Measures | MLE | Uniform prior | | | Jeffrey's prior | | | Gamma prior | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| **10** | Estimate | 0.5579 | 0.6137 | 0.5021 | 0.6409 | 0.5579 | 0.4463 | 0.5851 | 0.6303 | 0.5252 | 0.6560 |
| | MSE | 0.0415 | 0.0591 | 0.0309 | 0.0702 | 0.0415 | 0.0273 | 0.0492 | 0.0588 | 0.0297 | 0.0697 |
| **25** | Estimate | 0.5197 | 0.5404 | 0.4989 | 0.5507 | 0.5197 | 0.4781 | 0.530 | 0.5493 | 0.5086 | 0.5594 |
| | MSE | 0.0120 | 0.0142 | 0.0107 | 0.0156 | 0.0120 | 0.0103 | 0.013 | 0.0148 | 0.0107 | 0.0164 |
| **50** | Estimate | 0.5106 | 0.5208 | 0.5004 | 0.5259 | 0.5106 | 0.4902 | 0.5157 | 0.5255 | 0.5053 | 0.5306 |
| | MSE | 0.0056 | 0.0061 | 0.0052 | 0.0065 | 0.0056 | 0.0051 | 0.0058 | 0.0063 | 0.0053 | 0.0067 |
| **100** | Estimate | 0.5047 | 0.5098 | 0.4997 | 0.5123 | 0.5047 | 0.4946 | 0.5072 | 0.5122 | 0.5022 | 0.5147 |
| | MSE | 0.0027 | 0.0028 | 0.0026 | 0.0029 | 0.0027 | 0.0026 | 0.0027 | 0.0029 | 0.0026 | 0.0030 |
| **150** | Estimate | 0.5022 | 0.5056 | 0.4989 | 0.5072 | 0.5022 | 0.4955 | 0.5039 | 0.5072 | 0.5005 | 0.5089 |
| | MSE | 0.0017 | 0.0018 | 0.0017 | 0.0018 | 0.0017 | 0.0017 | 0.0018 | 0.0018 | 0.0017 | 0.0019 |
| **200** | Estimate | 0.5026 | 0.5051 | 0.5001 | 0.5064 | 0.5026 | 0.4976 | 0.5039 | 0.5063 | 0.5013 | 0.5076 |
| | MSE | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0013 | 0.0013 | 0.0013 | 0.0013 |

**Table 4. Average estimates (Estimates) and Mean Squared Errors (MSEs) for $\alpha = 2, \beta = 0.5, \theta = 0.5, a = 1$ and $b = 2$ under different priors, loss functions and sample sizes**

| n | Measures | MLE | Uniform prior | | | Jeffrey's prior | | | Gamma prior | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| **10** | Estimate | 2.2315 | 2.4546 | 2.0083 | 2.5638 | 2.2315 | 1.7852 | 2.3404 | 2.1523 | 1.7936 | 2.2402 |
| | MSE | 0.6640 | 0.9453 | 0.4945 | 1.1236 | 0.6640 | 0.4368 | 0.7873 | 0.3616 | 0.2776 | 0.4243 |
| **25** | Estimate | 2.0787 | 2.1618 | 1.9955 | 2.2030 | 2.0787 | 1.9124 | 2.1198 | 2.0664 | 1.9133 | 2.1043 |
| | MSE | 0.1918 | 0.2269 | 0.1711 | 0.2497 | 0.1918 | 0.1648 | 0.2074 | 0.1581 | 0.1393 | 0.1703 |
| **50** | Estimate | 2.0423 | 2.0832 | 2.0015 | 2.1035 | 2.0423 | 1.9606 | 2.0627 | 2.0391 | 1.9606 | 2.0586 |
| | MSE | 0.0891 | 0.0977 | 0.0838 | 0.1033 | 0.0891 | 0.0820 | 0.0929 | 0.0815 | 0.0755 | 0.0849 |
| **100** | Estimate | 2.0189 | 2.0391 | 1.9987 | 2.0492 | 2.0189 | 1.9785 | 2.0290 | 2.0181 | 1.9785 | 2.028 |
| | MSE | 0.0429 | 0.0450 | 0.0417 | 0.0463 | 0.0429 | 0.0413 | 0.0438 | 0.0411 | 0.0397 | 0.042 |
| **150** | Estimate | 2.0089 | 2.0223 | 1.9955 | 2.0290 | 2.0089 | 1.9821 | 2.0156 | 2.0086 | 1.9822 | 2.0152 |
| | MSE | 0.0279 | 0.0287 | 0.0275 | 0.0293 | 0.0279 | 0.0274 | 0.0283 | 0.0272 | 0.0267 | 0.0275 |
| **200** | Estimate | 2.0104 | 2.0205 | 2.0003 | 2.0255 | 2.0104 | 1.9903 | 2.0154 | 2.0102 | 1.9903 | 2.0152 |
| | MSE | 0.0203 | 0.0208 | 0.0200 | 0.0212 | 0.0203 | 0.0199 | 0.0206 | 0.0199 | 0.0195 | 0.0201 |

Jeffrey priors for a higher value of the shape parameter and these efficiency of the QLF is found to be consistent despite the differences in the sample sizes and the parameter values.

In general, the results in Tables 1-4 provide the averages of the MLEs (Mean estimates) and mean square errors (MSEs) for a shape parameter of the GoIED distribution. From the figures in Tables 1-4, it is shown that the average estimates tend to be closer to the true parameter value when sample size increases and the mean square errors (MSEs) all decrease as sample size increases which is in agreement with first-order asymptotic theory. It is noted that Bayesian estimators and maximum likelihood estimators (MLEs) become better when the sample size increases. However, for very large sample sizes this performance is observed to be the same for both methods.

## 8. CONCLUSION

In this article, we obtain Bayesian estimators for a shape parameter of Gompertz inverse exponential distribution. The Posterior distributions of this parameter are derived by assuming Uniform, Jeffrey and gamma prior distributions. Bayes estimators have been derived by using three loss functions under the three priors and posterior distributions respectively. The performance of these estimators have been assess on the basis of their mean square errors using the inverse transformation method of Monte Carlo Simulations for different parameter values and sample sizes. The results of the simulation and comparison show that using the *QLF* gives estimators with the smallest MSEs under all the prior distributions (gamma, Jeffreys and uniform). Most importantly, we found that Bayesian Method using Quadratic Loss Function (*QLF*) under Jeffrey prior produces the best estimators of the shape parameter compared to estimators using Maximum Likelihood method, Squared Error Loss Function (*SELF*) and Precautionary Loss Function (*PLF*) under both Uniform and gamma priors when the value of the estimated shape parameter is smaller whereas, Bayesian Method using Quadratic Loss Function (*QLF*) under gamma prior produces the best estimators of the shape parameter compared to estimators using Maximum Likelihood method, Squared Error Loss Function (*SELF*) and Precautionary Loss Function (*PLF*) under both Uniform and Jeffrey priors when the value of the estimated shape parameter is higher irrespective of the selected values of the parameters and the allocated sample sizes. It is also discovered that the values of the other two parameters of the GoIED have no effect on the estimators of the estimated shape parameter. In conclusion, this study considers a shape parameter of the Gompertz inverse exponential distribution and it encourages subsequent studies to look at the remaining two parameters of the distribution because in real life applications of a model it is necessary to understand the best method for estimating all the unknown parameters of the model.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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