Journal of Modern Physics, 2023, 14, 1735-1740
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Turbulence and Anomalous Transport

Bin Liang

College of Science, Chongqing University of Posts and Telecommunication, Chongqing, China
Email: liangbin@cqupt.edu.cn

How to cite this paper: Liang, B. (2023) Turbulence and Anomalous Transport. *Journal of Modern Physics*, **14**, 1735-1740.
https://doi.org/10.4236/jmp.2023.1413102

Received: November 5, 2023
Accepted: December 15, 2023
Published: December 18, 2023

Abstract

Turbulence is a thermodynamic system composed of a lot of vorticons rather than disturbance to random moving particles. The transport coefficients derived from the definition on turbulence show that the anomalous transport is a natural result of present turbulence in Tokamak plasma and provides an important theoretical reference for the design and operation of Tokamak.

Keywords

Turbulence, Vorticon, Anomalous Transport, Transport Coefficient

1. Introduction

The experimental results show that the transverse transport coefficient of Tokamak plasma is much greater than the neo-classical transport coefficient derived from the present theory, and this is called anomalous transport of Tokamak plasma. It is generally believed that turbulence causes anomalous transport in the Tokamak plasma, enhancing the loss of particles and heat. The study of turbulence is not only a difficult problem in fluid mechanics, but also an important practical problem, because the appearance of turbulence in Tokamak plasma is unavoidable, but now there is no unified theory that can explain the experimental data of anomalous transport. The starting point of previous studies on turbulence is to regard turbulence as the disturbance of random moving particles, and attempt to describe turbulence by nonlinear mathematical methods [1]-[8]. However, after more than half a century, the turbulence theory has failed to provide a statistical ensemble definition to compute average value of physical quantities, because the vortices in turbulence are not disturbances to random moving particles. Regardless of mass or energy, even small vortices are much greater than individual particles. Turbulence is a thermodynamic system composed of a lot of vorticons. The transport coefficients derived from the definition on turbulence show that the anomalous transport is a natural result of present turbulence in

Tokamak plasma and provides an important theoretical reference for the design and operation of Tokamak.

2. Vorticons in Turbulence

In 1852, Joule and Thomson discovered the cooling effect of throttling. In the 1970s, people made use of this effect to make micro refrigeration and widely used in scientific research and production activities. Why does the fluid temperature drop after passing through the orifice? It turns out that there must be turbulence in the fluid passed through the throttle hole, and the turbulence is composed of large and small vorticons, which reduce the temperature of the fluid. The explanation is as follows.

As shown in Figure 1, the X-axis is the interface of two flow layers with velocity u_2, u_1, whose direction is the direction of fluid. The circle centered at point O in the figure represents a medium element with mass m and rotational inertia J in the incompressible fluid, and point A is the contact point between the medium element and the interface.

The experimental results show that the tangential force acting on the unit interface area of the adjacent flow layers is [9]

$$f = \pm \eta \frac{\mathrm{d}u}{\mathrm{d}y}, \tag{1}$$

where $\frac{\mathrm{d}u}{\mathrm{d}y}$ is velocity gradient, η is coefficient of viscosity, the positive and negative signs indicate the tangential force acting on the flow layer with larger or smaller flow velocity, respectively. Suppose $u_2 > u_1$, the tangential force acting on micro-area s at point A is

$$F = -s\eta \frac{\mathrm{d}u}{\mathrm{d}y}, \tag{2}$$

the minus sign indicates the tangential force to the left. When point A is at rest instantaneously, the force acting on point O is the same as above, but in the opposite direction. Since the dimension of $\frac{\mathrm{d}u}{\mathrm{d}y}$ is the same as the frequency, write $\omega = 2\pi \frac{\mathrm{d}u}{\mathrm{d}y}$. Suppose the circle radius is r, so

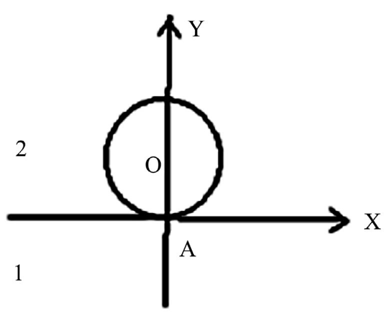

Figure 1. A medium element in fluid.

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t}=\frac{s\eta r}{2\pi}\omega . \tag{3}$$

Integrate the above equation and get

$$\omega=\omega_0\mathrm{e}^{\frac{s\eta r}{2\pi J}t} , \tag{4}$$

this is the angular velocity of the medium element in the vortex motion, where $\omega_0=2\pi\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_0$ is the initial angular velocity. It can be seen that as long as the flow velocity of flow layers is different, there will be viscous effect, and vortices will be produced. We call the medium element in the vortex motion the vorticon. A lot of vorticons constitute turbulence. The angular velocity of a three-dimensional vorticon is

$$\boldsymbol{\omega}=\boldsymbol{\omega}_0\mathrm{e}^{\frac{s\eta r}{2\pi J}t} , \tag{5}$$

where $\boldsymbol{\omega}_0=2\pi\sum\limits_{\substack{i,j,k=1,2,3\\ i\neq j\neq k}}\left(-\xi_{ijk}\right)\frac{\partial u_i}{\partial x_j}\boldsymbol{e}_k$, symbol ξ_{ijk} obeys the rules: $\xi_{123}=\xi_{231}=\xi_{312}=1$, otherwise it's minus 1.

In the absence of external fields (such as electromagnetic fields, gravitational fields, etc.), the kinetic energy of a vorticon is

$$\varepsilon=\frac{1}{2}mu^2+\frac{1}{2}J\omega^2 , \tag{6}$$

The first term on the right of the above equation is the translational kinetic energy of the vorticon, and the second term is the rotational energy.

We know that temperature is a measure of the average translational energy of molecules. As can be seen from the above equation if there is turbulence in a fluid in the advection state under adiabatic conditions, it means that a part of the molecular translational energy is transformed into the rotational energy of the vorticons, thus reducing the molecular translational energy of the fluid and lowering the fluid temperature. This is why where the Joule-Thomson throttling cooling effect occurs. Suppose the temperature of the unimolecular ideal gas is T and the temperature of the gas after turbulence is T'. According to the conservation of energy, there is

$$N\frac{3}{2}kT'+N_t\frac{3}{2}kT'=N\frac{3}{2}kT , \tag{7}$$

where N is the number of gas molecules and N_t is the number of vorticons, obviously there is $T'<T$.

It is well known that the molecules in an ideal gas do not interact with each other except collisions. There is no other interaction between vorticons in turbulence except viscous action and collision, and the viscous action has been reflected in the rotational energy of vorticons, thereby the turbulent can be regarded as an approximate ideal vortex system.

Assuming that the turbulent is in thermodynamic equilibrium and the Maxwell vorticon energy distribution law is

$$f(\varepsilon)=\frac{1}{\sqrt{2\pi}(kT)^{3/2}}\sqrt{\varepsilon}\mathrm{e}^{-\frac{\varepsilon}{2kT}}. \tag{8}$$

The above equation satisfies the normalization condition $\int_0^\infty f(\varepsilon)\mathrm{d}\varepsilon=1$. The average kinetic energy of vorticons in the turbulence is

$$\bar{\varepsilon}=\int_0^\infty \varepsilon f(\varepsilon)\mathrm{d}\varepsilon=3kT. \tag{9}$$

It is noted that the vorticon has 6 spatial degrees of freedom, so the above equation is consistent with the equipartition theorem of energy. The maximum probability energy of vorticons is kT. It can be seen that there are few vorticons with large energy in turbulent in thermodynamic equilibrium, and this is consistent with the actual situation.

The cooling of fluid accompanied by turbulence has a very important adverse effect on the stable operation of Tokamak plasma, this needs further study.

3. Anomalous Transport in Tokamak Plasma

The lateral drifted velocity of charged particles in electromagnetic field is [1]

$$\boldsymbol{\upsilon}_\perp=\frac{1}{B^2}(\boldsymbol{E}\times\boldsymbol{B}), \tag{10}$$

It's perpendicular to both the magnetic field and the electric field, and the magnitude is $E_\perp/B$.

Charged particles in electromagnetic fields may be also affected by non-electromagnetic force $\boldsymbol{F}$. Introducing an equivalent electric field

$$\boldsymbol{E}_{eff}=\boldsymbol{F}/q, \tag{11}$$

formula (10) can be written as

$$\boldsymbol{\upsilon}_\perp=\frac{1}{qB^2}(\boldsymbol{F}\times\boldsymbol{B}). \tag{12}$$

1) Diffusion coefficient

In Tokamak plasma, the Debye sphere with the radius equals Debye λ_D has weak electricity, so it is easy to form a particle cluster with the Debye sphere as the core, and the particle cluster is called the smallest vorticon. Since the electric potential $V(r)=\frac{q}{4\pi\varepsilon_0 r}\mathrm{e}^{-\frac{r}{\lambda_D}}$ of the Debye sphere is almost zero at $r=5\lambda_D$ [10], the smallest vorticon with radius $r=5\lambda_D$ is almost an electrically neutral cluster of particles, so the force on the smallest vorticon is mainly the plasma pressure

$$\boldsymbol{F}=-\frac{1}{n}\nabla p, \tag{13}$$

where n is the particle number density of the plasma. Substituting the above equation into Equation (12), since $\boldsymbol{B}\cdot\nabla p=0$ in plasma equilibrium, there is

$$\boldsymbol{\upsilon}_\perp = -\frac{1}{nqB}\nabla p \,. \tag{14}$$

Substituting $p = nkT$ into the above equation and supposing the temperature is uniform in space, the particle number flux across the magnetic field is obtained

$$n\boldsymbol{\upsilon}_\perp = -\frac{kT}{qB}\nabla n \,. \tag{15}$$

Writing the particle number flux as

$$n\boldsymbol{\upsilon}_\perp = -D_\perp^n \nabla n \,, \tag{16}$$

there is

$$D_\perp^n = \frac{kT}{qB} \,. \tag{17}$$

This is the diffusion coefficient caused by the gradient of particle number density. If $n = 10^{20}/\mathrm{m}^3$, $T = 10\ \mathrm{keV}$, Debye radius $\lambda_D = 7.4\times10^{-5}\ \mathrm{m}$, then the number of particles in the smallest vorticon is $n_D = \frac{4\pi}{3}(5\lambda_D)^3 n = 2.13\times10^{10}$, electric quantity $q = \frac{1}{2}n_D e$, the introduction factor 1/2 is due to the recombination of particles in the vorticon, thus obtaining

$$D_\perp^n = 50.64(T/B)\left(\mathrm{m}^2/\mathrm{s}\right)(\mathrm{keV}) \,. \tag{18}$$

This is similar to Bohm's empirical formula $D_B = 62.5(T/B)\left(\mathrm{m}^2/\mathrm{s}\right)(\mathrm{keV})$. Factor 1/16 was artificially introduced into Bohm's formula [11]. If the introduction is 1/20 rather than 1/16, then Bohm's formula becomes
$D_B = 50.0(T/B)\left(\mathrm{m}^2/\mathrm{s}\right)(\mathrm{keV})$. This is almost the same as Formula (18).

2) Coefficient of heat conduction

Supposing the particle number density is uniform in space, and as said before, the average energy of vorticons is 3*kT*, so the energy flux density across the magnetic field is

$$\left(\frac{n}{n_D}\right)3kT\boldsymbol{\upsilon}_\perp = -3k\left(\frac{n}{n_D}\right)\frac{kT}{qB}\nabla T = -\boldsymbol{\kappa}\nabla T \,. \tag{19}$$

Therefore, the coefficient of heat conduction of plasma due to temperature gradient is

$$\boldsymbol{\kappa} = -3k\left(\frac{n}{n_D}\right)\frac{kT}{qB} \,. \tag{20}$$

3) Coefficient of viscosity

The appearance of turbulence in Tokamak plasma indicates that there are flow layers with different velocity along the ring in the plasma. Supposing the velocity difference of smallest vorticons on both sides of a flow layer interface is Δu, then the viscous force along the interface is

$$F = \frac{n}{n_D} m_D \Delta u \upsilon_\perp = \rho \Delta u \upsilon_\perp \,, \tag{21}$$

where m_D is the mass of smallest vorticon, and ρ is the plasma mass density. Suppose the flow velocity difference Δu occurs within a radius of smallest vorticon on each side of the interface, *i.e.*

$$\frac{\Delta u}{l} = \frac{\mathrm{d}u}{\mathrm{d}r}, \tag{22}$$

where $l = 10\lambda_D$ is the diameter of smallest vorticon. Substitute the above equation into Equation (21), and get

$$F = \rho \upsilon_\perp l \frac{\mathrm{d}u}{\mathrm{d}r} \tag{23}$$

so we get the coefficient of viscosity

$$\eta = \rho \upsilon_\perp l . \tag{24}$$

Since Reynolds number $R_e = \rho \upsilon_\perp L/\eta$, the Reynolds number of Tokamak is

$$R_e = \frac{L}{10\lambda_D}, \tag{25}$$

where L is the maximum radius of cross-section of Tokamak plasma. That provides an important theoretical reference for the design and operation of Tokamak. For example, if the maximum radius of cross-section of Tokamak plasma is 1 m, Debye radius $\lambda_D = 7.4\times10^{-5}\ \mathrm{m}$, then Reynolds number $R_e = 1351$. It's below the critical Reynolds number but it's not too low.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Qin, Y.-W. (2011) The Physical Basis of Tokamak Experiments. Atomic Energy Press, Beijing, p. 14, 38, 46, 190, 197.

[2] Landau, L.D. and Lifshitz, E.M. (1984) Fluid Dynamics. Third Edition, Chapter Three.

[3] Pope, S.B. (2000) Turbulent Flows. Cambridge University Press, Cambridge.

[4] Sitenko, A. and Malnev, V. (1995) Plasma Physics Theory. Chapman & Hall, New York.

[5] Braginskii, S.I. (1965) Transport Processes in a Plasma. Reviews of Plasma Physics, Vol. 1, Consultants Bureau, New York, p. 205.

[6] Tsytovich, V.N. (1977) Theory of Turbulent Plasma. Consultants Bureau, New York & London.

[7] Connor, J.W. and Wilson, H.R. (1994) *Plasma Physics and Controlled Fusion*, **36**, 719. https://doi.org/10.1088/0741-3335/36/5/002

[8] Connor, J.W. (1988) *Plasma Physics and Controlled Fusion*, **30**, 619. https://doi.org/10.1088/0741-3335/30/6/001

[9] Liang, B. (2016) General Physics. Scholar's Press, Saarbrücken, p. 211.

[10] Li, D., *et al.* (2006) Plasma Physics. Higher Education Press, Beijing, p. 9.

[11] Okazaki, T. (2022) Fusion Reactor Design. University of Science and Technology of China Press, Hefei, p. 135. (In Chinese)