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Non-wave Solutions of the Maxwell-Einstein Equations

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Author's contribution

This whole work was carried out by the author YNZ.

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ABSTRACT

This article is devoted to treating of non-wave, i.e. instanton solution for the Maxwell-Einstein equations. Equations for the field of instanton and metric are derived. Metric of pseudo-Euclid space which is corresponding to transition between degenerate classical vacua of problem and is connected with presence at the space infinity divergent and convergent spherical electromagnetic waves is studied. An expression of the instanton is received and it's size is found. Value of pseudo-Euclid action is calculated. It is shown that instanton violates so called "week energetic condition" which is essential for space-time singularities proving.

Keywords: Instanton; pseudo-Euclid space; classical vacuum; pseudo-Euclid action.

1. INTRODUCTION

Gravitational instantons attract attention, starting from [1]. We recall the definition. Instantons are known as topologically nontrivial localized solutions of the classical pseudo-Euclidean field equations characterized by a finite action and connecting two different vacuums of the theory [2]. Euclidean version of the theory is introduced by replacing the Minkowski metric $g^{\mu\nu}(g^{00} = 1,\ g^{ij} = -\ \delta_{ij},\ i,j=1,2,3)$ to the Euclidean metric $\delta^{\mu\nu}$. Formally, the transition from the description in Minkowski space to a description in pseudo-Euclidean space is performed by replacing the time coordinate x^0 in Minkowski space to coordinate $y^0 = ix^0$ in pseudo-

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Euclidean space, while introducing a pseudo-Euclidean action Λ, associating it with the action in Minkowski space S by expression $\Lambda = iS$, $i = (-1)^{½}$.

Instantons of classical field equations in Minkowski space describe in the semiclassical approximation quantum tunneling process between degenerate classical states located near different classical vacuums. In the theory of Maxwell-Einstein (M-E) equations these degenerate states are states in which there is convergent (divergent) electromagnetic wave at spatial infinity, which represent the two degenerate vacuums of the theory. As was shown in [3] classical transition between these states is impossible. Indeed, if we consider the vacuum in which there is a convergent spherical electromagnetic wave (SEMW), and taking into account the curvature of space-time due to the wave then almost all the rays corresponding to small portions of the wave front will capture by curvature of metric and do not give a contribution to the outgoing wave[1]. Therefore, the role of the instanton of the M-E equations is extremely important in description of such an intuitive and "simple" phenomenon, which seems to be the process of transformation a convergent SEMW to a divergent one. Another important application of the instanton of the M-E equations is development a physical theory of electromagnetic resonators, which eliminates the unphysical singularities of fields, for example, in a spherical cavity [5]. And at last, an important application of the theory developed is cosmology, because the process of transformation of a convergent to a divergent SEMW is one of the main processes in the universe.

A brief scope of the present results is published in the Internet report [6].

2. BASIC EQUATIONS

As the initial equations we choose the Einstein's gravitational equations and the equations of the electromagnetic field in vacuum (Maxwell's equations) associated with each other [7]:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi K}{c^4} T_{ik}\,; F^{ik}_{,k} + \Gamma^l_{kl} F^{ik} = 0 \tag{1}$$

Here $R = R^i_i$– trace of Ricci's tensor R^i_k:, g_{ik}– metric tensor; T_{ik} and F^{ik}– tensor of energy-momentum and electromagnetic one; Γ^i_{kl} – Christoffel's symbols; c – light speed in vacuum, K – gravitation constant; indices i, k, l take values 0, 1, 2, 3; repeated indices mean summation; comma means usual, i.e. non-covariant derivative [8]. Let us find a solution of (1) which corresponds to existence of spherical light wave at $r \to \infty$. For this we use an expression for interval just as in well-known Schwarzschild problem [8]:

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \cdot d\varphi^2 \right) \tag{2}$$

$\nu = \nu(t, r, \theta)$, $\lambda = \lambda(t, r, \theta)$; $x^0 = ct$, t –time; $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ – spherical co-ordinates. SEMW is characterized by frequency ω and angular moment vector $\vec{J}$. Let us choose z - axis of the co-ordinate system in direction perpendicular to $\vec{J}$. It simplifies a treating because the dependence of azimuth angle φ in (1) may be omitted.

[1] *In other words, convergent wave is not focused to a point, or, in mathematical terms corresponding map is not homotopic to zero [4].*

Second equation in (1) may be transformed to [8]:

$$\frac{\partial}{\partial x^{\beta}}\left(\sqrt{-g}F^{\alpha\beta}\right)-\frac{\partial}{\partial x^{0}}\left(\sqrt{-g}F^{0\alpha}\right)=0$$
$$\frac{\partial}{\partial x^{\beta}}\left(\sqrt{-g}F^{0\beta}\right)=0,\alpha,\beta=1,2,3; \qquad (3)$$
$$\sqrt{-g}=e^{\frac{\lambda+\nu}{2}}r^{2}\sin^{2}\theta$$

where $\alpha =1, 2$ correspond for the SEMW of TM - type, and $\alpha=3$ – for SEMW of TE-type. Below we restrict ourselves with the case of TM - type[2] for nonzero components of electromagnetic tensor F_{ik}, taking into account that nonzero components of vector-potential A_i are only A_1, A_2:

$$F_{01}=\frac{\partial A_1}{\partial x^0},F_{12}=\frac{\partial A_2}{\partial x^1}-\frac{\partial A_1}{\partial x^2},F_{02}=\frac{\partial A_2}{\partial x^0} \qquad (4)$$

We use the Hamilton calibration $A_0 = 0$. For variables' separation we assume an additional conditions: $\lambda =\alpha(r, t)+\beta(\theta)$, $\nu = -\alpha(r, t)+\beta(\theta)$. Substitution (4) into (3) gives us two equations for the components A_1 and A_2:

$$e^{-\alpha(r,t)}\sin\theta\frac{\partial}{c\partial r}\left[r^{2}e^{-\beta(\theta)}\frac{\partial A_1}{\partial t}\right]+\frac{\partial}{c\partial\theta}\left[\sin\theta\frac{\partial A_2}{\partial t}\right]=0$$
$$\frac{\partial}{\partial\theta}\left[\sin\theta\left(\frac{\partial A_1}{\partial\theta}-\frac{\partial A_2}{\partial r}\right)\right]-e^{\alpha(r,t)}\frac{r^{2}}{c^{2}}\sin\theta\frac{\partial}{\partial t}\left[e^{-\beta(\theta)}\frac{\partial A_1}{\partial t}\right]=0 \qquad (5)$$

If we take a derivative of the first equation in (5) on *r* and second – on *ct,* then we exclude A_2 from the equations. Representing $F_{01}= \Psi(r,t)\cdot\Phi(\theta)$, we receive equations for $\Psi(r,t)$ and $\Phi(\theta)$:

$$\frac{d^{2}\Phi}{d\theta^{2}}+ctg\theta\frac{d\Phi}{d\theta}+l(l+1)e^{-\beta(\theta)}\Phi=0$$
$$e^{-\alpha(r,t)}\frac{\partial^{2}}{\partial r^{2}}\left(r^{2}\Psi\right)-e^{\alpha(r,t)}\frac{\partial^{2}}{c^{2}\partial t^{2}}(r^{2}\Psi)-l(l+1)\Psi=0 \qquad (6)$$

It leads from (6) that for $\beta \to 0$ $\Phi(\cos\theta) = P_l(\cos\theta)$, where $P_l(\cos\theta)$ – Legendre polinomial, and l is nonnegative integer[9].

Energy-momentum tensor's components T_k^i may be expressed by the components of metric tensor g_k^i with the help of Einstein's equation of gravity [8]:

[2] *A solution for the SEMW of TE – type does not need separate treating because Maxwell-Einstein equations so as energy-momentum tensor T_{ik} are invariant with the transformation $E \to -H$, $H \to E$.*

$$\frac{8\pi K}{c^4}T_k^i = R_k^i - \frac{1}{2}\delta_k^i R \tag{7}$$

where δ_k^i– unit 4-tensor, and R – is a trace of tensor R_k^i. The details of calculations one can find in [8], for example. Besides Christoffel's symbols presented in [8], we need some additional ones; a symbol ~ (tilde) means a derivative by the angle θ:

$$\Gamma_{12}^1 = \Gamma_{21}^1 = \frac{\tilde{\lambda}}{2}, \Gamma_{02}^0 = \Gamma_{20}^0 = \frac{\tilde{\nu}}{2}, \Gamma_{11}^2 = \frac{\tilde{\lambda}e^{\lambda}}{2r^2}, \Gamma_{00}^2 = \frac{\tilde{\nu}e^{\nu}}{2r^2}$$

A result looks as follows:

$$\begin{aligned}
&\frac{8\pi K}{c^4}T_0^0 = -e^{-\alpha-\beta}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2}+\frac{\tilde{\beta}}{2r^2}\left(2\tilde{\beta}+1\right)\\
&\frac{8\pi K}{c^4}T_1^1 = -e^{-\alpha-\beta}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2}-\frac{1}{r^2}\left[2\tilde{\tilde{\beta}}+2\left(\tilde{\beta}\right)^2+2\tilde{\beta}ctg\theta-\frac{\tilde{\beta}}{2}\right]\\
&\frac{8\pi K}{c^4}T_2^2 = \frac{1}{2}e^{-\alpha-\beta}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]+\frac{1}{2}e^{\alpha-\beta}\left(\ddot{\alpha}+\dot{\alpha}^2\right)+\\
&\frac{1}{2r^2}\left[\tilde{\tilde{\beta}}+\left(\tilde{\beta}\right)^2-\tilde{\beta}ctg\theta-\frac{\tilde{\beta}}{2}\right]\\
&\frac{8\pi K}{c^4}T_3^3 = \frac{1}{2}e^{-\alpha-\beta}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]+\frac{1}{2}e^{\alpha-\beta}\left(\ddot{\alpha}+\dot{\alpha}^2\right)-\frac{1}{2r^2}\left(\tilde{\tilde{\beta}}-\tilde{\beta}ctg\theta+\frac{\tilde{\beta}}{2}\right)\\
&\frac{8\pi K}{c^4}T_0^1 = -e^{-\alpha-\beta}\frac{\dot{\alpha}}{r};\frac{8\pi K}{c^4}T_0^2 = 0;\frac{8\pi K}{c^4}T_1^2 = -\frac{2\tilde{\beta}}{r^3}
\end{aligned} \tag{8}$$

We also express the components of energy-momentum tensor T_k^i through solution of Maxwell's equations for $F_{01} = \Psi(r, x^0)\Phi(\theta)$, $\Phi(\theta) = P_l(\cos\theta)$ - Legendre polynomial of order l [7]:

$$\begin{aligned}
&\frac{8\pi K}{c^4}T_0^0 = \frac{2K}{c^4}\left\{\frac{1}{2}e^{-2\beta}\Psi^2+\frac{1}{[l(l+1)]^2}\left[\frac{1}{2r^2}e^{-\alpha-\beta}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2+\frac{r^2}{2}e^{\alpha-\beta}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\}\Phi^2\\
&\frac{8\pi K}{c^4}T_1^1 = \frac{2K}{c^4}\left\{\frac{1}{2}e^{-2\beta}\Psi^2-\frac{1}{[l(l+1)]^2}\left[\frac{1}{2r^2}e^{-\alpha-\beta}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2+\frac{r^2}{2}e^{\alpha-\beta}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\}\Phi^2\\
&\frac{8\pi K}{c^4}T_2^2 = \frac{2K}{c^4}\left\{-\frac{1}{2}e^{-2\beta}\Psi^2+\frac{1}{[l(l+1)]^2}\left[\frac{1}{2r^2}e^{-\alpha-\beta}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2-\frac{r^2}{2}e^{\alpha-\beta}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\}\Phi^2\\
&\frac{8\pi K}{c^4}T_3^3 = \frac{2K}{c^4}\left\{-\frac{1}{2}e^{-2\beta}\Psi^2+\frac{1}{[l(l+1)]^2}\left[-\frac{1}{2r^2}e^{-\alpha-\beta}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2+\frac{r^2}{2}e^{\alpha-\beta}\left(\frac{\partial\Psi}{\partial x^0}\right)^2\right]\right\}\Phi^2\\
&\frac{8\pi K}{c^4}T_0^1 = -\frac{2K}{c^4}\frac{e^{-\alpha-\beta}}{[l(l+1)]^2}\frac{\partial\Psi}{\partial x^0}\left(\frac{\partial}{\partial r}r^2\Psi\right)\Phi^2;\frac{8\pi K}{c^4}T_0^2 = \frac{2K}{c^4}\frac{e^{-\beta}}{l(l+1)}\Psi\frac{\partial\Psi}{\partial x^0}\cdot\Phi^2\\
&\frac{8\pi K}{c^4}T_1^2 = \frac{2K}{c^4}\frac{e^{-\beta}}{l(l+1)}\frac{\Psi}{r^2}\frac{\partial}{\partial r}\left(r^2\Psi\right)\cdot\Phi^2
\end{aligned} \tag{9}$$

In accordance with [7] right sides of equations (9) is averaged over the angle *θ*. In addition, for the wave solutions they are averaged over time [7]. For the non-wave solutions the procedure of averaging over time has no meaning. Consider first the last three equations: for the T_0^1, T_0^2 and T_1^2. Note, that their right hand sides, except the equation for the T_0^1 are of the order of value ~ $r_s^2/r^2 << 1$, where $r_s^2 = K<f>^2/2c^4$, <*f*>- solution's order of value, $f = r^2\Psi$. Therefore, at distances of the order of the wavelength of light right hand sides of equations can be omitted[3]. This is consistent with the equations for T_0^2 and T_1^2(5), if $\tilde{\beta} = 0$. The equation for T_0^1 in (9) we will use to find *α*.

3. TREATMENT THE EQUATIONS

Subtracting the second equation in (9) from the first one and equating the result with the similar operation which is done with equations (8) we receive:

$$\frac{2K}{c^4}\left[e^{-\alpha}\left(\frac{\partial}{\partial r}r^2\Psi\right)^2 + e^{\alpha}\left(\frac{\partial}{\partial x^0}r^2\Psi\right)^2\right] = \frac{e^{\beta}\left[l(l+1)\right]^2}{\Phi^2}\left[\tilde{\tilde{\beta}} + 2\tilde{\beta}^2 + \tilde{\beta}\left(ctg\theta + \frac{1}{4}\right)\right] = A \tag{10}$$

A is a constant. The solutions of (6), corresponding to the equation (10) one can treat in pseudo-Euclidean space which metric follows from the Minkowski space's metric with substitution time co-ordinate x^0 to "imaginary time" co-ordinate $-iy^0$ in pseudo-Euclidean space. At the same time one can introduce pseudo-Euclead action *Λ*, which is connected with the action *S* in Minkowski space as follows $\Lambda = iS$, $i = (-1)^{½}$. It is known [2] that localized solutions of Euclidean field equations with finite Euclidean action are instantons. Instantons of classical field equations in Minkowski space describe in quasi-classical limit tunneling between degenerate classical states, which contain convergent or divergent SEMW. This procedure turns second hyperbolic equation in (6) to the elliptic one. If one suppose its finiteness at $r \to \infty$ then he receives a condition $A = 0$ from the equation (10). This provides second equation (6) looks as follows:

$$e^{\alpha} = \pm\frac{\partial f}{\partial r}\left(\frac{\partial f}{\partial y^0}\right)^{-1};$$
$$f'' + \left(\frac{f'}{\dot{f}}\right)^2\ddot{f} \mp \frac{l(l+1)}{r^2}\frac{f'}{\dot{f}}f = 0; \tag{11}$$
$$f = r^2\Psi$$

Here prime still means a derivative on $x^1 = r$, and point – on $y^0 = c\tau$.

Signs ± hereafter correspond to different vacuums of the theory, located at $\tau \to \pm\infty$. Using (11) we can rewrite the equations for instantons (8) and (9) in the form:

[3] *See details in [3].*

Einstein equations:

$$-e^{-\alpha}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2}=\frac{K}{c^4}\frac{f^2}{r^4}$$
$$\frac{1}{2}e^{-\alpha}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]-\frac{1}{2}e^{\alpha}\left(\ddot{\alpha}+\dot{\alpha}^2\right)=-\frac{2K}{c^4}\left\{\frac{f^2}{2r^4}+\frac{1}{[l(l+1)]^2}\frac{e^{\alpha}}{r^2}\left(\dot{f}\right)^2\right\} \quad (12)$$

Point here and below means derivative y^0.

Maxwell equations:

$$f''+e^{2\alpha}\ddot{f}-e^{\alpha}\frac{l(l+1)}{r^2}f=0 \quad (13)$$

Consider the equations (12). They are compatible if the following condition is true:

$$e^{\alpha}\left(\ddot{\alpha}+\dot{\alpha}^2\right)\mp\frac{2\dot{\alpha}}{r}=\frac{K}{c^4}\frac{1}{r^3}\frac{df^2}{dr} \quad (14)$$

Condition (14) allows us to rewrite equations (12) in the form:

$$-e^{-\alpha}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2}=\frac{K}{c^4}\frac{f^2}{r^4}$$
$$e^{-\alpha}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]=-\frac{2K}{c^4}\left[\frac{f^2}{2r^4}+\frac{1}{r^3}\frac{df^2}{dr}\right]$$

(15)

Treating these equations is very difficult even numerically. Therefore, we are interested mainly in the asymptotic behavior of their solutions at distances $r \geq \lambda$, λ is a wavelength of light. Note that their right sides are of the order $\sim r_s^2/r^2 \ll 1$ and they can be omitted with the adopted accuracy. Then Einstein's equations reduce to a single equation. Its solution gives the metric in a space free of matter which is well known.

$$e^{\alpha}\approx 1+\frac{const}{r} \quad (16)$$

Here the value of the constant *const* is to be determined. Furthermore, this asymptotic equation has auto-scale solutions, depending on $z = c\tau/r$. For such solutions instead of (14) we obtain the equation

$$\sigma'' \mp 2\frac{\sigma'}{\sigma} = 0, \sigma = e^{\alpha}, \sigma' = \frac{d\sigma}{dz} \quad (17)$$

Equation (17) is easily integrated and leads to the expression $Ei(\alpha) = \pm 2z$, where *Ei*– integral exponent. Using the well-known series expansion [10]:

$$Ei(\alpha) = \ln|\alpha| + \sum_{k=1}^{\infty} \frac{\alpha^k}{k \cdot k!}$$

we can get the expression for the metric for large values of *z*:

$$e^{\alpha} \approx 1 + e^{-2|z|} \quad (18)$$

Formula (18) describes the transition between the vacuum states with flat metric corresponding to the presence of at $z \to -\infty$ convergent SEMW, and at $z \to +\infty$ divergent one. This transition is localized with respect to *τ*, the size of the region of localization is a ~ *r/c*. Einstein's equations are also satisfied because "time" *τ* does not appear in them, and the equation (17) has a solution that can be represented for small *z* (large *r*) as a series expansion

$$\sigma = e^{\alpha} = 1 + \mu z - \mu z^2 + \frac{\mu(\mu+1)}{3} z^3 + \ldots \approx 1 + \frac{\mu\tau}{r}, \mu = \sigma'(0) \quad (19).$$

4. PSEUDO-EUCLIDEAN ACTION

Let us calculate the action in curved space-time [7]

$$S_f = -\frac{1}{16\pi c}\int F_{ik} F^{ik} \sqrt{-g}\, d\Omega;$$
$$d\Omega = dx^0 dx^1 dx^2 dx^3;$$
$$\sqrt{-g} = r^2 e^{\beta(\theta)} \sin\theta \quad (20)$$

Turning to action in pseudo-Euclidean space $\Lambda = iS_f$, $dx^0 = -icd\tau$, and using (11) and the normalization condition for $\Phi(\theta)$ [9], we receive (if $\beta = 0$)

$$\Lambda = \frac{1}{4c^2}\int\left\{\left(\frac{\partial A_r}{\partial\tau}\right)^2 \pm \frac{2r^2}{c[l(l+1)]^2}\frac{\partial^2 A_r}{\partial r\partial\tau}\frac{\partial^2 A_r}{\partial\tau^2}\right\} r^2 dr d\tau \quad (21)$$

In (21) is also taken into account that A_θ can be expressed through A_r [7]. Let us treat extremes of Λ. To do this we calculate the variation Λ on A_r provided condition $\delta A_r = 0$ on the boundaries of integration and equate it to zero. As a result, we obtain:

$$\delta\Lambda = \frac{1}{2c^2}\int \delta A_r \frac{\partial^2}{\partial \tau^2}\left\{A_r \pm \frac{r^2}{c[l(l+1)]}\frac{\partial^2 A_r}{\partial r \partial \tau}\right\} r^2 dr d\tau = 0 \tag{22}$$

Because of the arbitrariness *δA*$_r$ integrand in (22) is equal zero, which gives the equation of the instanton:

$$\frac{\partial A_r}{\partial \tau} \pm \frac{r^2}{c[l(l+1)]^2}\frac{\partial^3 A_r}{\partial r \partial \tau^2} = 0 \tag{23}$$

It reduces to the equation:

$$z\mathrm{Y}'' \mp [l(l+1)]^2 \mathrm{Y} = 0$$
$$\mathrm{Y} = \frac{\partial A_r}{\partial \tau}, z = \frac{c\tau}{r} \tag{24}$$

The solution of this equation has the form:

$$\mathrm{Y}(z) = \sqrt{z} Z_1\left(2l(l+1)\sqrt{\mp z}\right) \tag{25}$$

Z_1- cylindrical function. Below, however, we will use another solution of equation (24), since the solution *Y(z)* does not have finite action. Calculating the pseudo-Euclidean action for the instanton $A_r^I(r,\tau)$, we receive

$$\Lambda\left(A_r^I\right) = \frac{3}{4c^2}\int \left(\frac{\partial A_r^I}{\partial \tau}\right)^2 r^2 dr d\tau > 0 \tag{26}$$

Pseudo-Euclidean action must be calculated for a classical trajectory which beginning and end ($\tau \to \pm\infty$) lie in the regions where the space-time is not curved, i.e. at $r \to \infty$, where the field of SEMW tends to zero. Among the set of solutions of equation (23) satisfying this condition, we choose the solution:

$$\frac{\partial A_r^I}{\partial \tau} = cE \exp\left\{\mp \omega\tau - \frac{c}{\omega}[l(l+1)]^2 \frac{1}{r}\right\} \tag{27}$$

where *E* - is a constant with the dimension of the electrical field. Its value is related to the so-called topological charge of the instanton

$$Q = \int_{-\infty}^{\infty} \frac{\partial A_r^I(r=\infty,\tau)}{\partial \tau} d\tau = 2E\frac{c}{\omega} \tag{28}$$

In evaluating the integral in (28) we use the expression (27)[4]. An important feature of the solution (27) is that it field decreases at $r \to 0$, which is consistent with the tunneling nature of the instanton.

In calculating the pseudo-Euclidean action for the solution (27) to avoid divergence of the integral in (26) we cut off the integral over *dr* in the upper limit at the distance r_0, having a sense of the size of the instanton, which will be defined below. With this in mind, the result of calculations (26) looks as follows:

$$\Lambda(A_r^I) = 6\frac{E^2 r_c^3}{\omega}[l(l+1)]^3 K(l)$$
$$K(l) = \int_{x_0}^{\infty} \frac{e^{-x}}{x^4} dx, x_0 = 2\frac{r_c}{r_0} l(l+1) \tag{29}$$

Recall that the action $\Lambda(A_r^I)$ determines the probability *w* of transition of convergent SEMW to divergent one: $w \sim \exp\left(\Lambda(A_r^I)\right)/\hbar$, $\hbar = h/2\pi$, h – is the Plank constant [2].

The value r_0 we will find from the condition of matching metrics inside and outside the instanton. In the outer region metric is given by [3].

$$e_{out}^{\alpha} = \left[1 - \frac{r_c}{r} + \left(\frac{r_s}{r}\right)^2\right]^{-1} \tag{30}$$

Metric in the inner region is found from formula (11), where we substitute the solution (27), given that $f = ir^2/c\partial A_r/\partial\tau$. The result looks as follows:

$$e_{in}^{\alpha} = \frac{2r_c}{r}\frac{1}{l(l+1)} + \left(\frac{r_c}{r}\right)^2 \tag{31}$$

Given that $r_s << r_c$ and given in equation $e^{\alpha}_{in} = e^{\alpha}_{out}$ most significant members we get (up to terms ~ $[l(l+1)]^{-1}$)

$$r_0 = r_c\left[1 - \frac{2}{3l(l+1)}\right]^{-1} \tag{32}$$

[4] *Typically, the integral in (28) is normalized to the right side of the equation that leads to values of Q = 1 (iinstanton) and Q = -1 (anti-instanton) [11].*

Let us calculate the amount of $R^{00} = \frac{8\pi K}{c^4} T^{00}$, using (8) for the metric (31) where $T^{00} = W$ – field energy density of the instanton.

$$R^{00} = g^{00} R_0^0 = e^{\alpha}\left[\frac{1}{r^2} + e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right)\right]; e^{\alpha} = e_{in}^{\alpha} \tag{32}$$

The result of calculation is shown in Fig. 1

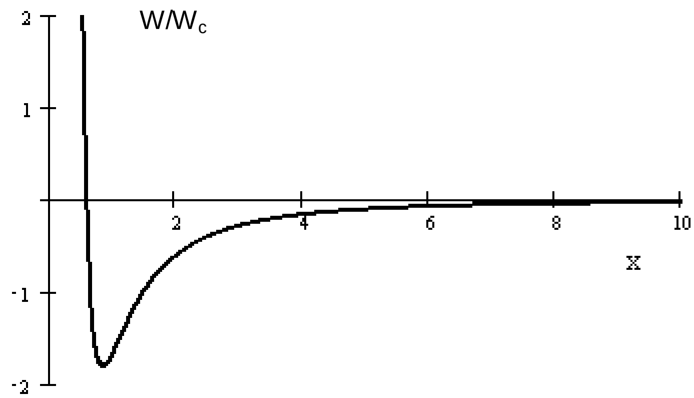

Fig. 1. Field energy density of the instanton W(*x*); W$_c$=$\frac{c^4}{8\pi K r_c^2}$, *x = r/r$_c$*, *l = 3*.

Qualitative behavior of W (x) is stood the same for any l

The calculation shows that near the boundary of the instanton and wave, i.e. for $x \approx x_0$ $(r \approx r_0)$ energy density is negative. The calculation shows also that the magnitude $R_{00} < 0$ in a sufficiently large vicinity of $x = x_0$ for all *l*. The latter circumstance is essentially for the proof of the presence (or rather lack of it) of singularities, which, as is known, is based on the fact $R_{\alpha\beta}\,\xi^{\alpha}\,\xi^{\beta} > 0$, where ξ – is any non space-like *4* – vector[5] [12]. Absence of singularities associated with horizons of the metric (30), can be seen both from the expression (31) and from Fig. 1. The only fatal singularity is a singularity at $x = 0$, where $W(x) \sim x^{-4}$.

5. PROBLEM OF INSTANTONS FROM ENERGETIC POINT OF VIEW

During the propagation of SEMW part of its energy converts into other energy forms, such as energy of the gravitational waves. This issue was left outside the scope of the work (see [3,5,7].

In the literature there are different points of view on the question on interaction of EMW and gravitational waves. In [13] the author argues that the processes of transformation of the two photons in the graviton (and back) are prohibited by the conservation laws. At the same

[5] *For proving of $R_{\alpha\beta}\,\xi^{\alpha}\,\xi^{\beta} < 0$ one can take, for example, ξ (1, 0, 0, 0).*

time, Wheeler did not rule out such a possibility [14]. In [15], these processes are considered without any discussion. These differences can be overcome, if we consider the photon-graviton processes in the presence of a static gravitational field created by SEMW, which removes the restrictions imposed by the conservation laws[6]. Leaving this issue for further discussion, make the following remark. Consider the first relation from (11).

$$e^{\alpha} = \pm c \frac{\partial f}{\partial r}\left(\frac{\partial f}{\partial \tau}\right)^{-1}; \; f = r^2\Psi, \tau = i\frac{x^0}{c} \tag{33}$$

for instantons, and a similar relation for waves [7]

$$e^{\alpha} = \pm \frac{\partial f}{\partial r}\left(\frac{\partial f}{\partial x^0}\right)^{-1}; \; f = r^2\Psi \tag{33a}$$

binding metrics and fields of instanton or electromagnetic waves. For definiteness we take in (33) and (33a) "+" sign. Using Maxwell's equations in curved space-time [8] one can write them in the form, respectively.

$$\begin{aligned} e^{-\frac{\alpha}{2}} &= 1 - i\int_r^{\infty} \frac{(rot\vec{H})_r}{\vec{E}_r}\,dr \\ e^{-\frac{\alpha}{2}} &= 1 - \int_r^{\infty} \frac{(rot\vec{H})_r}{\vec{E}_r}\,dr \end{aligned} \tag{34}$$

$\vec{E}, \vec{H}$ - electric and magnetic fields of the instanton or SEMW. The magnitude of the integrand is proportional to the conductivity (the role of the current density plays displacement current density), the value for which is real for wave and imaginary for instanton. The first means that the energy irreversibly transfers from SEMW to some other form, which is most likely connected with gravitational waves. The second indicates the reversible transfer of energy from the electromagnetic wave to the instanton, with subsequent return to the SEMW.

A question of interest is that, at what stage of the study was the neglect of gravitational waves, and what role they play in the problem. If we argue by analogy with the problem of the gravitational collapse of a non-spherical body, it can be assumed that the emission of gravitational waves will accompany the propagation of a spherical electromagnetic wave with a nonzero *l*, that, in the end of ends allow to speak about a spherically symmetric metric for $l \neq 0$. Thus, used in this work, as well as in [3,5,7], averaging tensor T_i^k (9) on the angle θ as a consequence led to the fact that gravitational waves have been left out of consideration.

6. CONCLUSION

This article is devoted to treating the role of instantons in considering the dynamics of spherical electromagnetic waves by means of Maxwell-Einstein equations. Due to instantons

[6] *Like that the diagrams with three free ends become possible in quantum electrodynamics [16]*

convergent wave can be transformed into a divergent one what allow transmission of information from the past to the future. This article discusses the two different solutions for instantons – an auto-scaled one depending on $z = c\tau / r$ (25) which does not have a finite pseudo-Euclidean action, and the solution (27) with the finite one $\Lambda(A_r^l)$ (29). A special feature of the first solution is that in a world where it could be realized, the past is separated from the future with an infinite barrier, i.e. there is no flow of time in this world. The second solution is more consistent with the state of affairs in the real world - past goes to the future with some finite probability.

The result obtained above, consisting in violation by instantons of the so-called "weak energy condition" $T_{\alpha\beta}\ \xi^{\alpha}\ \xi^{\beta} > 0$, where ξ – is any non space-like 4 – vector is important in research of the space-time singularities [13].

Note that most of the work on gravitational instantons available on the resource [17], are devoted to the classification of instanton solutions of Maxwell-Einstein in multidimensional Riemannian manifolds and their applications to the physics of black holes.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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